Reinventing the redundant target paradigm to distinguish serial and parallel processing of written words

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ABSTRACT

The visual system can encode many stimuli across the visual field simultaneously, but the number of objects that can be fully identified in parallel is limited. At the extreme, some objects might have to be identified serially. One useful tool for distinguishing parallel from serial processing is the redundant target paradigm, which compares responses to displays containing one target to displays containing two targets. Many parallel models predict a positive redundant target effect: faster correct responses to two targets. Here we revisit the redundant target paradigm by developing and testing predictions for a standard self-terminating serial model that accounts for the accuracy as well as the speed of each response. Surprisingly, it predicts *slower* responses to two-target displays than one-target displays. To test that prediction, we conducted three experiments that each measured performance for three different judgments of written words: color detection (detecting colored letters), lexical decision (detecting real words among pseudowords), and semantic categorization (detecting nouns that refer to living things). In all the experiments, only the color detection task yielded a positive redundant target effect, while the lexical and semantic tasks yielded zero or negative effects. These results are consistent with low-level features (color) for two stimuli being processed in parallel, while the meanings of two words are processed serially. Altogether, this study informs models of reading and furthers the development of general theories of response time that include errors.

Key words: redundant targets; divided attention; serial and parallel processing; response time; visual word recognition.

The study of perception has long been animated by the question of whether multiple stimuli can be processed in parallel, or whether strict processing capacity limits require serial processing. Here we re-examine that question in the context of visual word recognition: can two words be recognized simultaneously? This is an important question because competing models of natural reading disagree as to whether multiple words are processed in parallel during each gaze fixation (Engbert et al., 2005; Reichle et al., 2006; Reilly & Radach, 2006; Snell & Grainger, 2019b). To investigate whether it is possible to recognize two words simultaneously, we use the *redundant target paradigm.* This experimental paradigm differs from natural reading, assessing instead how well participants can process two words at once when they are encouraged to try. We compare task performance to the predictions of serial and parallel processing. This complements other approaches that use dual-tasks or measure spatial attention effects (Johnson et al., 2022; White et al., 2018, 2020; White, Palmer, et al., 2019).

Fundamentals of redundant target effects

The redundant target paradigm grew out of a larger visual search literature to investigate whether observers can process multiple stimuli presented simultaneously at different visual field positions (van der Heijden, 1975). The observer's task is to view a display and report the presence or absence of stimuli that belong to a target category. Non-target stimuli are termed "distractors." On some trials, one target is presented. On other trials, multiple targets are presented simultaneously – that is, the display contains redundant targets. The *redundant target effect* is a speeding of correct response times on trials with multiple targets compared to trials with a single target. Such an effect, also termed a "redundancy gain," can be taken as evidence that the targets were processed in parallel.

Studies that have used the redundant target paradigm can be divided into two broad categories. Studies in the first category seek to distinguish between a parallel model and a serial model (van der Heijden, 1975). They compare response times between displays that consist of two (or more) targets, versus displays that contain one target and no other stimuli. Studies in the second category seek to distinguish between two flavors of parallel models: those with separate activations caused by each stimulus, versus those with interactive "coactivations" (C. W. Eriksen et al., 1989; Miller, 1982; Mordkoff & Yantis, 1991). To do so, response times are compared between displays that contain two targets and displays that contain one target and one distractor. These "mixed" trials are not useful for testing the serial model, which is the focus of the present study.

Therefore, we focus on the comparison between displays with a single target presented alone and displays with two targets. As shown in **Figure 1**, contrasting predictions for correct response time arise from a standard unlimited-capacity parallel model and a standard serial model. They are called "standard" models because of strong assumptions about the independence of the processes for each stimulus. Both standard models assume that search is *self-terminating*: the observer responds as soon as they detect a target. The parallel model assumes that when two targets are present, they are independently processed in separate channels that race to produce the response. The completion time of each process is variable across trials. On two-target trials, the response time is determined by the faster of the two processes, so the observer is faster on average than when only one target is present (Raab, 1962). Thus, the parallel model predicts a positive redundant target effect: a speeding of correct responses.

The standard serial model, in contrast, assumes that one stimulus is processed at a time (Townsend & Nozawa, 1995; van der Heijden, 1975). If the first target of two simultaneously presented targets is correctly identified, then the response time is on average the same as when only one target is present—the redundant target has no effect on performance. Previous descriptions of this serial model stop there; but as we explain in our new theory section below, a serial model that incorporates errors predicts a *slowing* of correct response times if the first target to be processed is misidentified, and search continues to process the second target correctly.

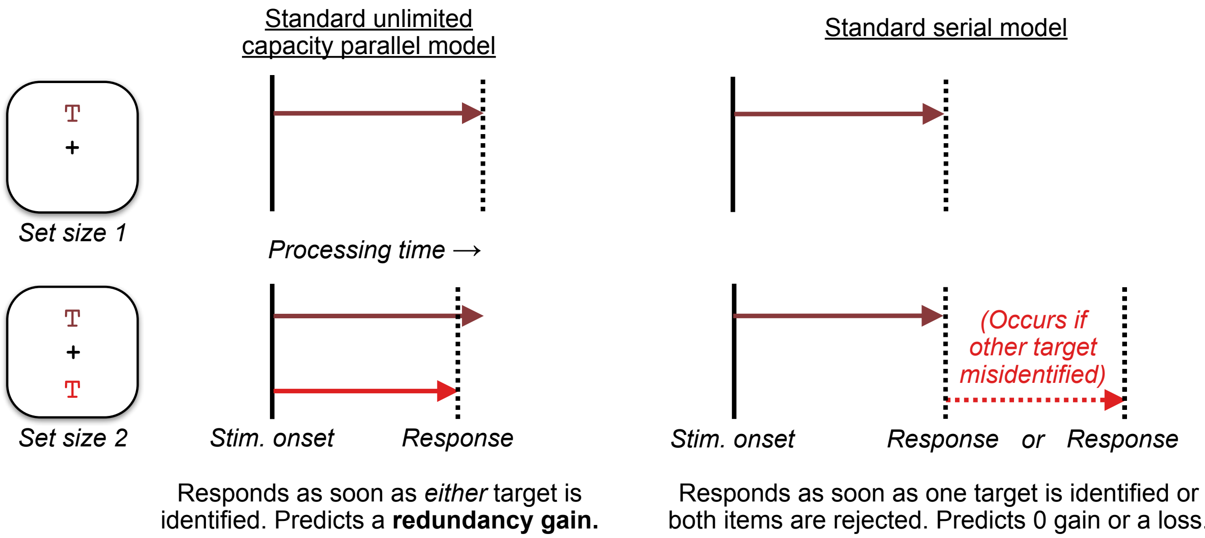

Figure 1: Diagram of a standard parallel and a standard serial model processing displays containing one target (set size 1) or two targets (set size 2). The parallel model predicts faster correct responses for set size 2 because the response is triggered by whichever process happens to finish sooner. The serial model predicts either zero effect of set size, or a loss (slower responses to set size 2).

Redundant target effects have been used to reject the standard serial model for processing simple visual features, such as detecting lights, discriminating colors, orientations, and motion directions (Corballis, 2002; Donkin et al., 2014; Egeth et al., 1989; Ridgway et al., 2008; Schwarz, 2006; Thornton & Gilden, 2001). Redundant target effects have also been found with auditory stimuli (e.g. (Schröter et al., 2007) and with bimodal stimuli (e.g. (Gondan et al., 2010; Hershenson, 1962). Face recognition has also been studied with redundant target effects (Fitousi, 2021).

Letters are an interesting case, being the building block of words. Several studies have used tasks that require the participant to distinguish one target letter from other letters. When the task uses a "go-no/go" design—to press a button when a target is detected and otherwise make no response—there are positive redundant target effects (e.g. Grice & Reed, 1992; Mordkoff & Yantis, 1991; van der Heijden et al., 1983). That is also true when the observer makes a vocal response ("yes" or "no"; van der Heijden, 1975). However, other studies have found no redundant target effect when the procedure is slightly different, such as requiring a choice response on each trial (Fournier & Eriksen, 1990; Grice & Reed, 1992; van der Heijden et al., 1983). One possibility is that letters are processed in parallel, producing a positive redundant target effect, but that effect can be masked by later decision- or response-selection processes when the response rule is more complicated. In other words, if conditions increase the degree of limited capacity, then it can overcome the redundant target effect.

Redundant target effects for word recognition tasks

We now turn to the central topic of this article: redundant target effects for written words. Such effects could reveal the extent to which higher-level semantic or linguistic information about two stimuli can be processed in parallel. A handful of studies have gone down that road, with mixed results. They have differed in four important respects: whether the redundant targets in a single trial are identical words; whether the single-target trials also contain a 'filler' stimulus; what the task is (semantic categorization vs. lexical decision), and how the subject responds (go/no-go vs. choice).

We first summarize studies that used a lexical decision task, in which the targets are real English words and the distractors are meaningless pseudowords. In Mullin & Egeth (1989), words were presented above and below fixation. The task was to make a go/no-go response to the presence of a word. Trials either contained 1 pseudoword, 1 real word, 2 pseudowords, or 2 real words (a "pure" design with no mixed pairs). In one experiment (their Experiment 2), the redundant targets were identical words. In that case, there was a significant redundant target facilitation of response times. A similar result was reported by Egeth et al., (1989), and, with words present to the left and right of fixation, by Hasbrooke & Chiarello (1998) and Mohr and colleagues (Mohr et al., 1994, 1996). However, a redundant target effect for identical words might be explained by facilitation at a sub-lexical level (Abrams & Greenwald, 2000)[1]. Mullin & Egeth (1989)'s third experiment included a condition with two real word targets that were different from each other (also in a lexical decision task). In that experiment, the redundant target effect was significantly *negative*, meaning that response times were slowed by the addition of a second target.

A second set of word recognition studies have used semantic categorization tasks. In Mullin and Egeth (1989)'s first and fourth experiments, one or two words were presented above and below fixation. The task was a go/no-go response to the presence of a word belonging to a given semantic category (e.g., 'animals'). Notably, each of the four categories contained only 5 words. In one experiment, when two targets were presented they were identical words, and in the other experiment the two targets were different words. There was no significant redundant target effect in either experiment, consistent with the serial model.

The second relevant study using semantic categorization was by Shepherdson & Miller (2014). We focus on their Experiment 3. Stimuli were presented to the left and right of fixation. On each trial, the subject had to make a yes/no response to report the presence of a word belonging to a target semantic category. Most importantly, the "single-target" trials also contained a "filler" stimulus that was a pseudo-word. They found an advantage for the redundant target condition compared to this modified baseline. Interpreting this experiment is difficult. The critical question is whether a serial, self-terminating model predicts no effect between these conditions. Such a prediction holds only if the target is always processed first by the serial process and the filler pseudoword is never processed first, which seems unlikely. Thus, we conclude this experiment should be considered along with experiments that used a mixed-trial baseline to test co-activation models and not as a test of the serial, self-terminating model.

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[1] Nonetheless, presenting copies of the same word across the field provides a redundancy gain that might help patients with macular degeneration to read, with rapid serial visual presentation (Snell et al., 2022)

In summary, the effects of redundant targets in word recognition tasks require further investigation. The presence of a redundant target effect might depend on the subject's task (lexical decision or semantic categorization), the mode of response (go/no-go or forced-choice), and on whether the experiment includes trials in which a target is paired with a distractor. In the new experiments reported below, we investigate all those factors, and compare lexical and semantic tasks to a font color task. In doing so we also test new models of parallel and serial processing that consider both response time and accuracy.

Response time and accuracy in redundant target effects

Most of the redundant target studies reviewed above focused on only correct response times and use tasks in which accuracy is near ceiling. Other studies have focused on accuracy, for instance in the context of spatial summation (e.g., Robson & Graham, 1981; Verghese & Stone, 1995). One important study compared redundant target effects on accuracy and response time (Mordkoff & Egeth, 1993). This work has shown that typical parallel models predict positive redundant target effects for accuracy as well as response time.

As shown in the following section on our new theory, serial processing of the individual stimuli can lead to a *negative* effect of redundant targets on response time. This hinges on the possibility of errors: if the first target to be processed is misidentified as a distractor, then search continues, and the second target may be correctly identified. Those correct responses increase the mean response time for two-target displays compared to correct responses to single target displays. Thus, our new theory explicitly considers the accuracy of each stimulus recognition process when predicting response times.

New Theory

The new theory generalizes previous models of pure response time by adding the possibility of errors (misclassifying targets as distractors or vice versa). The **Appendix** contains full mathematical descriptions of three classes of models. Here we describe them in intuitive terms and emphasize the qualitative redundant target effects that they each predict.

Consider the standard self-terminating serial model developed for response time (Townsend & Nozawa, 1995). Like others of its type, it assumes discrete component processes for each stimulus. In addition, it allows time for residual processes before a response is made that do not depend on the stimulus. This model has been called standard because it includes a number of independence properties (see Appendix). We add to this model the possibility of an error and additional independence assumptions concerning the errors and the relationship between error and response time.

Our goal in building this new theory is to compare the *qualitative* predictions of the various models: whether they predict positive, negative, or zero redundant target effects on response time and accuracy. Our goal is not to quantitatively fit models to our data; that is a larger endeavor (never before attempted for redundant target effects) which we leave for the future. For now, it is sufficient to generate qualitative predictions that allow experimental data to rule out some models.

Figure 2 illustrates the typical range of predicted redundant target effects for each class of model. The standard serial model always predicts negative effects. The unlimited-capacity parallel model always predicts positive effects. The fixed-capacity parallel model can predict either negative or positive effects. These results of the new theory are described in more detail in the following paragraphs.

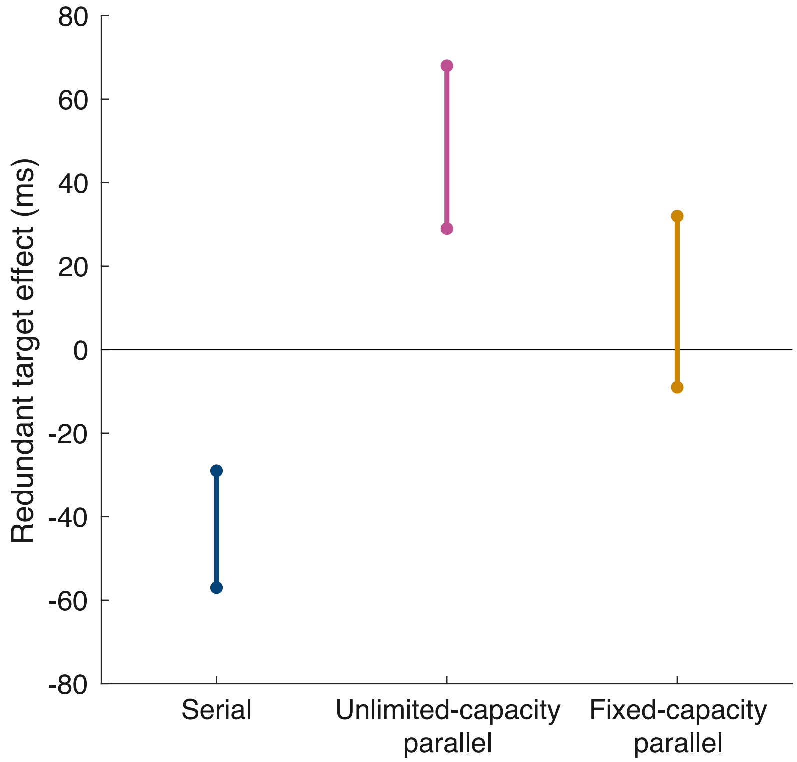

Figure 2: Model predictions. For each type of model, we plot the typical range of redundant target effects on correct response times. For all models we assume that for single-target trials, errors occur on 5% of trials and the mean correct response time is 800 ms. Each model's range encompasses the smallest and largest effects that we could generate with the parameters described in the Appendix.

Predictions of Our Serial Model

The most important result concerns the predictions made by our new standard self-terminating serial model with errors. Specifically, it predicts a *negative* redundant target effect. This is unlike the corresponding pure response time model that predicts no effect of redundant targets. The source of this different result has to do with what happens when there is an error in processing the first stimulus; if the first target to be processed is mis-identified as a distractor, processing continues for the second stimulus. If this second target is correctly identified, then this correct response time is included in the analysis with the other trials in which the first target was processed correctly. Thus, the correct response times for the redundant target condition are a mixture of two cases:

those trials in which the first target was correctly identified and the response was made quickly, and those trials in which the first target was not correctly identified and processing continued to the second target. This new result makes the serial model with errors more distinctive from the parallel models, in terms of correct response times. The predictions for errors are discussed below.

Quantitatively, this model's predicted redundant target effect is given by Equation 5 in the Appendix, which is repeated here:

$$\mu_{t,correct} - \mu_{tt,correct} = -\frac{(1-p_t)}{(2-p_t)} E\left[\boldsymbol{D}_{t,incorrect}\right].$$

The redundant target effect is the difference between the mean correct response time for a single target ($\mu_{t,correct}$) and the mean correct response time for two targets ($\mu_{tt,correct}$). This effect depends on only two factors, the probability correct on single-target trials, p_t, and the mean component processing time for a single target when the participant makes an error (a 'miss' response), $E\left[\boldsymbol{D}_{t,correct}\right]$.

Crucially, the serial model always predicts a negative redundant target effect. An illustration of this prediction is in **Figure 2.** For this illustration, we assume that for single-target trials, accuracy p_t is 0.95 (5% errors) and the mean correct response time is 800 ms. The upper end of the range is predicted with the assumption that the mean component processing time for an error is equal to the mean component processing time for a correct response. The lower end of the range is predicted with the assumption that the mean component processing time for an error is twice that for a correct response. See the Appendix for more detail.

Predictions of Our Unlimited-Capacity, Parallel Model

Our second result concerns the standard, self-terminating, unlimited-capacity parallel model with errors. The corresponding model without errors predicts a positive redundant target effect. We show that this generalizes to models with errors. Errors can reduce the size of the effect, but it always remains positive. Thus, there remains a sharp contrast in the predictions for this parallel model and the standard self-terminating serial model.

The predictions of this parallel model are given by Equation 11 in the Appendix:

$$\mu_{t,correct} - \mu_{tt,correct} = \left(\frac{1}{2-p_t}\right) E\left[\boldsymbol{D}_{t,correct}\right] - \left(\frac{p_t}{2-p_t}\right) E\left[\min\left\{\boldsymbol{D}_{t_1,correct}, \boldsymbol{D}_{t_2,correct}\right\}\right]$$

Here, p_t is the probability correct on single-target trials, $E[\mathbf{D}_{t,correct}]$ is the mean correct component processing time for a single target, and $E[\min\{\mathbf{D}_{t1,correct}, \mathbf{D}_{t2,correct}\}]$ is the mean of the minimum of the two component processing times when two targets are presented and judged correctly.

The predicted effect is always positive. This is primarily because the model's response to two targets is driven by whichever of the two stimulus processes finishes first, hence the "min" function in the equation above. On average this is faster than the response to a single target. The equation also makes clear why the predicted effect is always positive: it is the difference between two products, and the first is always larger. This must be the case, as is clear when examining each part of the two products separately. First:

$$\left(\frac{1}{2-p_t}\right) \geq \left(\frac{p_t}{2-p_t}\right)$$

because $0 \leq p_t \leq 1$. Second:

$$E\left[\boldsymbol{D}_{t,correct}\right] > E\left[\min\left\{\boldsymbol{D}_{t_1,correct}, \boldsymbol{D}_{t_2,correct}\right\}\right]$$

because the mean difference between two identically distributed (non-negative) variables is always less than the mean of one of those variables alone.

An illustration of this prediction is the middle bar in Figure 2. For this illustration, we used predictions of two specific models described in the Appendix. The upper point is for a diffusion model with parameters that generate large redundant target effects. The lower point is for a linear ballistic accumulator model with parameters that generate relatively small redundant target effects. While not strict limits, these model outputs illustrate the range of predictions from the standard, self-terminating, unlimited-capacity parallel model. They are always positive.

Predictions of Our Fixed-Capacity, Parallel Models

The parallel model that can most mimic a serial model is one that has limited capacity. The limited capacity slows processing when there are two stimuli and thus reduces and possibly eliminates the redundant target effect. Unfortunately, this model is so general that it does not make very specific predictions. Here, we consider a special case of the limited-capacity parallel model: the fixed-capacity, parallel model. The idea of fixed capacity is that a set of parallel processors extract the same total amount of information from multiple stimuli as they do from a single stimulus. Thus, splitting a fixed set of 'resources' between multiple stimuli introduces a cost. Most of the prior work with this model has been in the domain of accuracy (Scharff et al., 2011; Shaw, 1980; White et al., 2018).

We investigated two special cases of self-terminating fixed-capacity parallel models in which we assume a particular stochastic process for each stimulus being processed. Predictions of these two special cases define the range of redundant target effects plotted in Figure 2 (rightmost bar). First, with a diffusion process of sensory evidence accumulation (Palmer et al., 2005), the model yields positive redundant target effects on correct response times, for all relevant parameter values (as well as a positive effect on accuracy). This prediction defines the upper end of the range of effects predicted by the fixed-capacity parallel model in Figure 2. However, with a linear ballistic accumulator process (Brown & Heathcote, 2008), the fixed-capacity, parallel model can predict a *negative* redundant target effect on correct response time (a slowing), despite a positive effect for accuracy. This is illustrated in Figure 2 by the lower end of the range of predicted effects for the fixed-capacity model. In essence, because of fixed capacity, the addition of a second target slows processing of both stimuli. Thus, among many parallel models that generate positive response time effects of redundant targets, there are models with fixed-capacity limits that yield the opposite result.

Thus, there is an asymmetry in using the redundant target paradigm to test the serial model and fixed-capacity, parallel models. All our models are assumed to be standard, self-terminating models. A positive redundant target effect rejects the serial model, but a negative redundant target effect does not reject all possible fixed-capacity, parallel models. Thus, the redundant target paradigm is a good test for rejecting both the serial model and the unlimited-capacity parallel model but not the fixed-capacity parallel model. Nevertheless, it is relevant to distinguishing serial and parallel models. Indeed, many experiments with simple feature tasks have used this test to rule out the standard serial model (e.g., van der Heijden, 1975).

Predictions About Errors

Our last result concerns the usefulness of models that incorporate errors. All of the models described above predict a positive redundant target effect on accuracy (that is, fewer errors on trials with 2 targets than on trials with 1 target). This is not a surprise for typical parallel models that have been investigated in summation experiments of accuracy alone (Graham et al., 1978). What is new is that this result also occurs for our serial model, even though that model predicts slower response times for two targets. The reason is that when two targets are present and processed sequentially, there are two chances to correctly detect target presence, so accuracy increases compared to when only 1 target is present – even though doing so takes more time on average. While this result for errors does not distinguish between the serial and parallel models, it introduces a result that is specific to errors and that is not accounted for by pure response time models.

Summary of our experiments

We conducted three experiments that differed in two factors. The first factor was how participants responded to the stimuli. "Go/No-Go" is a procedure that requires the participant to press a button if they see a target and to make no response if they see no targets. This is a common, simple procedure for redundant target effects. "Choice" is a procedure that requires the participant to press one of two buttons to categorize each stimulus display. This is the most common procedure in the larger visual search literature. As discussed above, some prior research suggests that a go/no-go procedure is more sensitive for detecting redundant target effects (Grice & Reed, 1992). Previous studies about word recognition have used a mix of go-no/go and choice procedures, so we used both in different experiments.

The second factor we manipulated was whether the words presented on two-word trials were "correlated." In the "correlated" design, the two words were either both targets or both distractors. In the "uncorrelated" design, there also were trials in which one target was paired with one distractor. The correlated design maximizes the fraction of trials that test the serial model (one target alone vs two targets), but introduces contingencies that might affect performance (Mordkoff & Yantis, 1991). The uncorrelated design requires more trials but is more typical in visual search generally. The inclusion of mixed trials might affect the participant's strategy and encourage them to process both stimuli, thus we use it in Experiment 3 to compare to the correlated design. Altogether, these variations in procedure span the range of tasks used in prior redundant target studies.

Notably, in all three experiments, when two words were present, they were always two different words. This differs from some previous redundant target experiments that used identical words on two-target trials (Mullin & Egeth, 1989) and might produce effects due to sub-lexical facilitation.

In each experiment, we also measured performance in three different tasks (color detection, lexical decision, and semantic categorization). The *color task* required participants to judge a low-level visual feature of the words, and served as a control condition for which we expected positive redundant target effects. The *lexical decision task* requires the subject to distinguish real English word targets from pseudoword distractors. The *semantic categorization task* requires categorizing words either as targets that belong to a category of “living things” and distractors that belong to a category of “non-living things”. The semantic and lexical tasks might tap into different levels of linguistic processing, and have both been used in prior redundant target studies (Egeth et al., 1989; Mullin & Egeth, 1989). In sum, within each of our three experiments, we carry out a side-by-side comparison of redundant target effects that arise in three tasks using the same stimuli. The tasks differ in which they require low-level color feature detection, lexical access, or semantic categorization. Altogether, this study includes over 184,000 trials of data from a total of 257 participants.

Experiment 1: Go/No-Go procedure with correlated stimuli

Methods

Participants: Participants were recruited from around the world using Prolific (www.prolific.co, accessed August 2021-May 2023). Participants gave informed consent in accordance with the Declaration of Helsinki and Barnard College’s Institutional Review Board. All participants indicated being fluent speakers who learned English as their first language, with no literacy difficulties, and normal or corrected-to-normal vision. For each task, we aimed to recruit an independent sample of 28 participants, half male and half female. That sample size was chosen on the basis of a power analysis of an independent pilot data set, seeking at least 95% power to detect a redundant target effect on response time of 15 ms.

Experiment	Task	N recruited	N excluded	N included	N Female	Mean age [min max]
1: Go/No-Go, correlated	Color	30	2	28	13	32 [19 50]
	Lexical	28	0	28	12	32 [20 47]
	Semantic	28	0	28	14	30 [20 47]
2: Choice, correlated	Color	29	0	29	16	27 [18 50]
	Lexical	28	0	28	14	29 [19 48]
	Semantic	28	0	28	18	27 [19 50]
3: Choice, uncorrelated	Color	28	0	28	22	20 [18 24]
	Lexical	29	1	28	15	31 [20 48]
	Semantic	29	1	28	14	31 [20 48]

Table 1: number of subjects in each experiment and task, as well as the ages in years of the included subjects.

Table 1 indicates the number of subjects and exclusions for all experiments in the study. Across all experiments in this study, our criteria for exclusion were: overall proportion correct less than 0.6, or proportion correct less than 0.5 in more than 1 block of 60 trials. In Experiment 1, two subjects in the color task were excluded for the latter reason: one of them had 3 blocks with accuracy less than 0.5, and the other had 4 such blocks (out of 10 blocks). This was a risk of the go/no-go task conducted over the web browser: if the participant gets distracted mid-block, the experiment carries on without them.

Stimuli: We created and presented stimuli with PsychoPy 3 (Peirce et al., 2019), run through the web browser using Pavlovia (https://pavlovia.org/). Each stimulus size and position were defined as a fraction of the height of the participant's screen; thus, the dimensions in degrees of visual angle likely varied across participants. Participants were asked to sit with their head roughly 1 arm's length from their screen. A central black fixation cross, 4.5% of screen height in width, was present throughout each trial except during feedback. The stimuli consisted of letter strings, of length between 4 and 6 letters.

The word lists are described below and provided fully in the public data repository (https://osf.io/a9kqj/). They were drawn in Courier font, with the height of an "o" or "x" occupying 3.8% of the screen height. That is roughly 0.7 degrees visual angle on a typical laptop. The specific words (or pseudowords) and font color varied across tasks, as described below.

Trial sequence: An example trial is illustrated in **Figure 3A**. Each trial began with just the fixation mark present for 750 ms. Then either 1 or 2 words were presented for 183 ms. There were two possible word positions, centered horizontally and either just above or just below the fixation mark. The distance from the fixation mark to the center of each word was 10% of the screen height (roughly 1.8 degrees visual angle on a typical laptop at arm's length). About two letter o's would fit stacked vertically in the empty space between the screen center and the words.

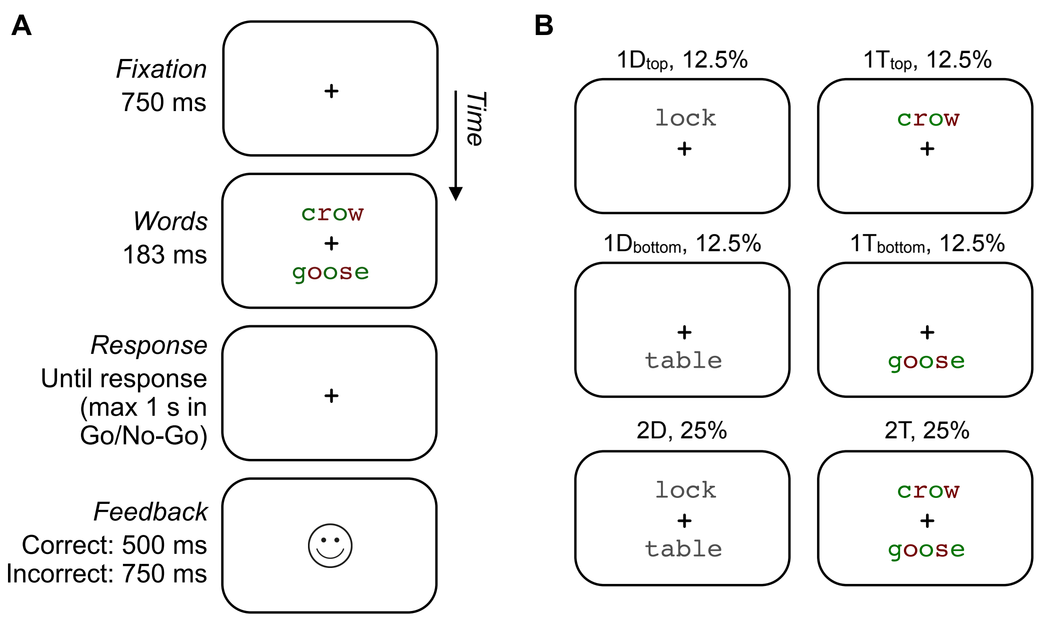

Figure 3: Stimuli and Design. (A) Example trial sequence with two color targets. **(B)** Examples of the trial types for the color task in Experiments 1 and 2. The text above each panel indicates the percentage of trials that were of that condition. "D" = distractor, "T" = target.

The trials were evenly distributed between these 4 conditions: 1 target, 1 distractor, 2 targets, and 2 distractors. **Figure 3B** illustrates examples of each trial type, and Table 2 lists the proportion of trials assigned to each combination of stimuli at the top and bottom locations. Each location could have no word ('None'), a distractor word, or a target word. When just 1 word was present, it was equally likely to be in the top or bottom location. Unlike in Experiment 3, there were never any mixed pairs of 1 target and 1 distractor. Thus, in this experiment, the words in each display were "correlated," meaning that when there were 2 words present, they were either both distractors or both targets.

After the words disappeared, the participant was free to respond. In these go/no-go tasks, the participant was instructed to press the spacebar as soon as they detected a target, and to do nothing if they saw no targets. Up to 1 second was allowed for a response. After the response interval elapsed or was ended by a keypress, feedback was given: the fixation cross was replaced with a smiley face for 500 ms if the response was correct, or a neutral face for 750 ms if the response was incorrect. Then, the fixation cross reappeared and another trial began (except when it came time for a break between blocks, see below).

		Bottom word		
		None	*Distractor*	*Target*
Top word	*None*	*N/A*	*0.125*	*0.125*
	Distractor	*0.125*	0.25	0
	Target	*0.125*	0	0.25

Table 2: The probability of stimulus pairings at the top and bottom locations in Experiments 1 and 2. The word at each location was either absent, a distractor, or a target. The green shading highlights conditions when 2 words were present. In this design, the two words were perfectly correlated, meaning that they were either both targets or both distractors.

Procedure**:** Once they accessed the experiment in Pavlovia, participants read a consent form and indicated their acceptance by pressing a key to continue. The program advanced through four pages of instructions with example stimuli. Then the participant conducted practice trials, which continued for at least 50 trials until the participant had responded correctly to 36 of the most recent 40 trials. Having completed that, they began the main experimental trials which came in 10 blocks of 60 trials each. Before starting the first block, participants were reminded to keep their head 1 arm's length from the screen, maintain central fixation, and to respond as quickly as possible without making unnecessary errors. Between each block they were given written feedback about their percent accuracy (*P*) and the opportunity to rest. If *P* for the most recent block was at least 96%, the feedback said, "Very nice! You got {*P*}% correct. In the next block, try to go a bit faster, while still getting at least 90% correct." If *P* <= 72% correct, the feedback said, "Good job. You got *P*% correct. In the next block, try to get above 90% correct." Otherwise, the feedback simply said, "You're doing great!" Participants completed the whole experiment in roughly 30 minutes, on average.

Color detection task: Each word was either drawn in all dark gray letters (RGB 79, 79, 79 out of 255) or its letters alternated between dark red (RGB 115, 18, 18) and dark green (RGB 17,102,15). Targets were defined as words written in colored letters; distractors were words written in gray letters. The words were drawn from the same set as in the semantic categorization task (see below).

Lexical decision task: All the letters were dark gray (RGB 79, 79, 79). There were a total of 246 items in the stimulus set, half real English words and half pronounceable pseudowords. Within both categories, 33 had 4 letters, 46 had 5 letters, and 44 had 6 letters. The real words were all nouns that were also used in the color & semantic tasks, with mean lexical frequency 16.4 occurrences per million (ranging 0.3-391). The pseudowords were generated using MCWord (Medler & Binder, 2005) to have trigram statistics (the probability of any sequence of three letters) matched to real words. Across the 600 trials in the experiment, each word was repeated on average 3.7 times. We took care to match the mean lexical frequency and word lengths across trials with 1 real word and trials with 2 real words.

Semantic categorization task: All the letters were dark gray as in the lexical task. There were a total of 246 English nouns in the stimulus set, half of which referred to living things and half to non-living things. Within the living category there were 39 4-letter words, 42 five-letter words, and 42-six-letter words. They referred to animals (e.g., "bird," "turtle", "woman") and plants (e.g., "fern", "orchid"; and one was "fungus"). The non-living category had 37 4-letter words, 42 five-letter words, and 44-six-letter words. They referred to common household items (e.g. "towel"), pieces of clothing ("e.g. "shoe"), and types of buildings (e.g., "cabin"), as well as natural non-living things (e.g., "snow"). The distributions of lexical frequencies in the living and non-living categories were highly overlapping, with means 19.7 and 14.5, respectively. Each word was repeated on average 3.7 times within the experiment.

Analysis: We computed two measures of performance in each condition: the mean response time on correct trials, and the percent of trials with incorrect responses (errors). In most cases we focus on trials with targets, because only those measures test our models that assume self-terminating search for targets. For both measures, we compared the means on trials with two targets to trials with one target with paired t-tests. All t-test p-values were corrected for false discovery rate across the 9 tests done for each measure in the entire study (Benjamini & Hochberg, 1995). We also used bootstrapping to get a 95% confidence intervals (CI) of each mean difference. Lastly, we supplement our pairwise tests with Bayes factors (BFs), which quantify the strength of evidence (Rouder et al., 2009). The BF is the ratio of the probability of the data under the alternate hypothesis (that two means differ) relative to the probability of the data under the null hypothesis (that there is no difference). A BF of 10 would indicate that the data are 10 times more likely under the alternate hypothesis than the null. BFs between 3 and 10 are regarded as substantial evidence for the alternate hypothesis, and BFs greater than 10 as strong evidence. Conversely, BFs between 1/3 and 1/10 are considered substantial evidence for the null hypothesis, etc. We computed BFs using the bayesFactor toolbox by Bart Krekelberg (https://doi.org/ 10.5281/zenodo.4394422).

To compare across tasks across experiments, we also fit linear mixed effect (LME) models to single-trial data, with fixed effects of the task, the set size (number of words), random intercepts and slopes by participant, and random effects for individual stimulus items. All p-values for a certain test were corrected for false discovery rate across the 9 tests done in the study.

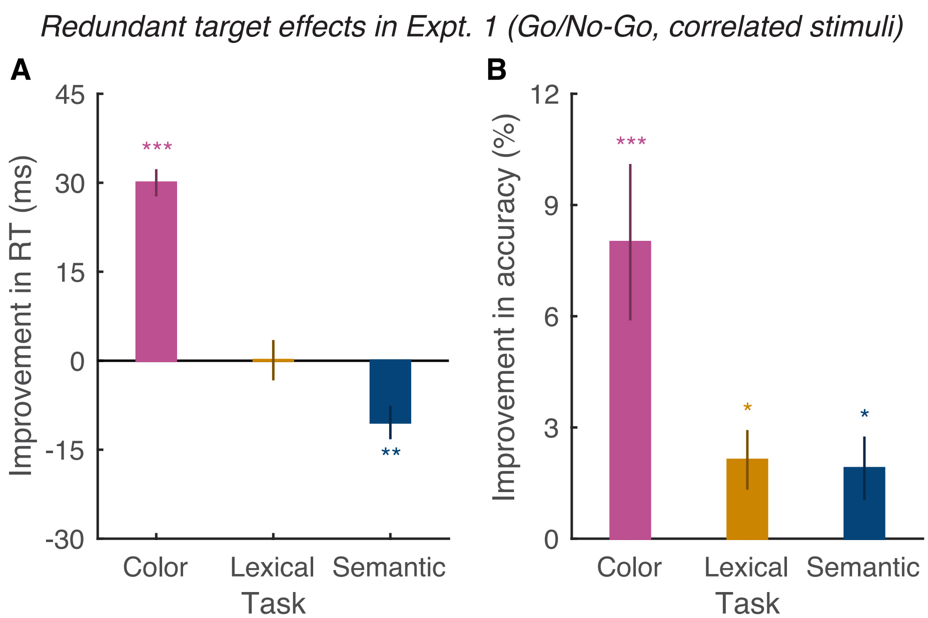

Figure 4: Redundant target effects in Experiment 1. These bar plots show the mean *improvement* in (**A**) mean correct response time (RT) and in (**B)** accuracy for 2 targets compared to 1 target. The mean performance levels from which these difference scores were derived are in Figure 5. Error bars are ± 1 SEM. Asterisks indicate that the mean effect is significantly different from 0 (***p<0.001, **p<0.01, FDR-corrected).

Results

<u>Response times</u>: **Figure 4A** shows that in the color task, there was a positive redundant target effect: a speeding of correct responses to two targets compared to one, by 32 ms on average. However, in the semantic task, there was a significantly negative

effect (a slowing of responses) of 11 ms on average. In the lexical decision task, there was no effect of redundant targets. **Table 3** lists the statistics on each effect. The mean response times in each individual condition (rather than the differences between one and two targets) are shown in the top row of **Figure 5.**

To compare the redundant target effects across tasks, we also fit three linear mixed-effect models to single-trial correct response times (target-present trials only). Compared to the color task, both the lexical and semantic tasks had significantly different redundant target effects (both $F>36$, $p<10^{-8}$). The lexical and semantic tasks yielded effects that were marginally different ($F(1, 15475)=3.86$, $p=0.056$).

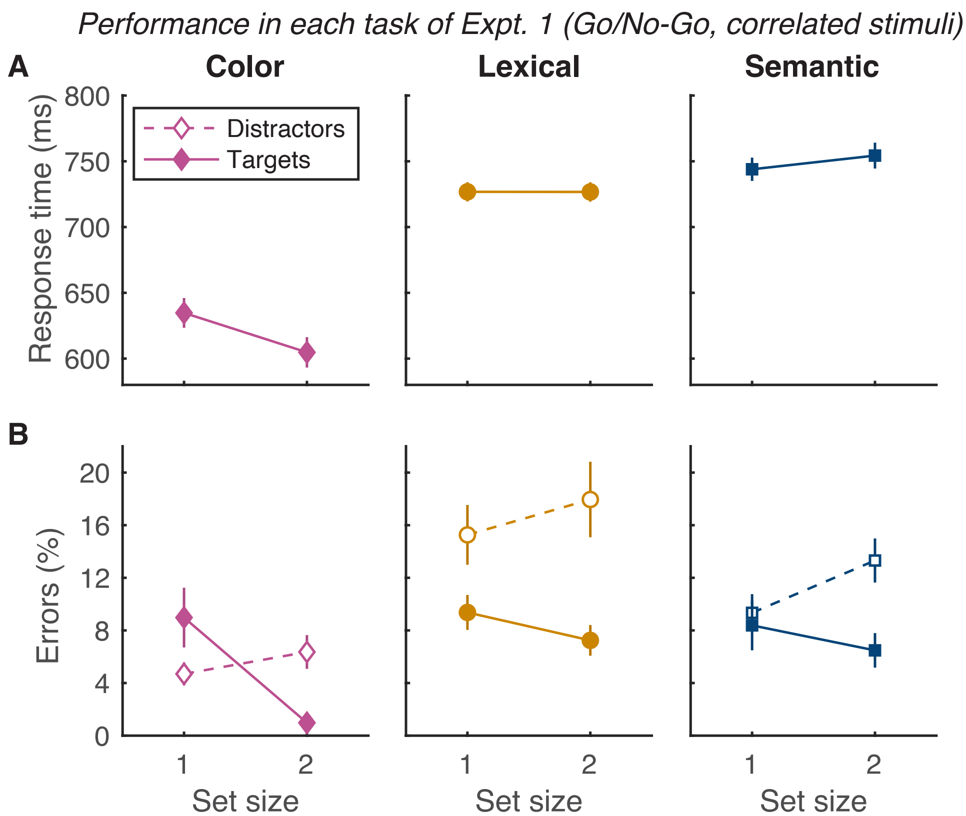

Figure 5: Mean performance in each task of Experiment 1, plotted separately for targets and distractors, for set size 1 vs. 2. **(A)** Mean correct response times. Note that there is no correct response time data for distractors, because in these go/no-go tasks, the correct response to distractors was to not press any key. **(B)** Percent of trials with errors. Data for distractors are plotted with open symbols and dashed lines (showing how often the participants made false alarms). Error bars are ± 1 SEM.

Accuracy: **Figure 4B** shows the mean improvements in accuracy caused by a redundant target. There was a significant improvement in all three tasks (see statistics in Table 3). The mean percentage of trials with errors in each condition (including for trials with distractors) are plotted in the bottom row of **Figure 5**. Compared to the color task, both the lexical and semantic tasks had smaller redundant target effects on accuracy for detecting targets (both $F>39$, $p<10^{-9}$). The effects did not differ significantly between the lexical and semantic tasks ($F(1, 18573)=0.75$, $p=0.50$).

Task	**Effect mean**	**Effect SEM**	**95% CI**	***t***	***p***	**BF**
	Correct response time (ms)					
Color	31.98	2.49	[27 36]	12.67	6.35×10^{-12}	1.09×10^{10}
Lexical	-0.11	3.53	[-6 8]	0.03	0.98	0.20
Semantic	-10.67	3.53	[-17 -6]	3.60	0.002	27.69
	Errors (percent)					
Color	7.95	2.10	[4.54 13.23]	3.72	0.001	35.778
Lexical	2.12	0.80	[0.62 4.02]	2.59	0.017	3.22
Semantic	1.90	0.86	[0.49 4.12]	2.19	0.038	1.54

Table 3: Statistics on redundant target effects in Experiment 1, describing the mean *improvement* in response time or error rate, contrasting 1-target displays vs. 2-target displays. The degrees of freedom for the t-tests was 27. For each measure (response time or accuracy), p-values are corrected for false discovery rate across all 9 comparisons including all 3 experiments in the study. BF = Bayes Factor.

Discussion

The redundant target effects in the first experiment suggest that the colors of the letters within two words can be processed in parallel, leading to speeding of response times. However, the meanings of the two words are not necessarily processed in parallel. This is because the presence of a second word target in the lexical decision task yielded 0 improvement in response time, and the semantic categorization task yielded a significant slowing of response time.

Our new theory shows that such a negative response time effect is in fact consistent with the standard serial model, even when accompanied by an increase in accuracy (see Appendix). It can be explained by participants occasionally mis-categorizing the first target they process as a distractor, then going on to correctly process the other target, with a slower response time compared to correct trials with single targets. It is potentially interesting that this negative redundant target effect was significant in the semantic task but not the lexical task, but we lack strong statistical evidence that those two results were significantly different from each other.

It is also noteworthy that redundant targets improved accuracy in all three tasks (although that effect was significantly larger in the color task than in the other two). Is that evidence of parallel processing of two words in all three tasks? Not necessarily: the serial model also predicts improvements in accuracy that go along with slowing of response speeds. This is merely a statistical facilitation: there are two chances to reach the correct decision when two targets are present. Also note that, as shown in the bottom row of Figure 5, errors on trials with distractors *increase* when the set size is 2 compared to 1 (a decrease of accuracy, opposite to the pattern for trials with targets). This might also be consistent with a shift in decision criterion, as participants are somewhat more likely to report "target present" when they see two words than when they see one word.

Thus, the entire data set is necessarily consistent with parallel processing in only the color task, which yielded a positive redundant target effect in both response time and accuracy.

Experiment 2: Choice procedure with correlated stimuli

Experiment 1 assessed redundant target effects with the simplest possible design ("correlated stimuli", meaning no trials with mixed targets and distractors) and the procedure thought to be most sensitive (go/no-go). In the next two experiments, we used variations of the paradigm that have previously been used to study word recognition and might alter the participant's strategy. In Experiment 2, we used the same "correlated" stimulus conditions as Experiment 1, but we required participants to make a choice response (target present vs absent) on every trial.

Methods

Participants: Participants were recruited in the same way as in Experiment 1. See Table 1 for counts. Applying the same accuracy criteria as in Experiment 1, no participants had to be excluded.

Stimuli and procedure: All methodological details were the same as in Experiment 1, except the participant had to make a categorization judgment on every trial: press the left arrow if no target was present on the screen, or the right arrow if any targets were present on the screen. The response interval was unlimited, but participants were requested to "respond as quickly as you can without making unnecessary errors."

Analysis: To eliminate outliers, trials were excluded with response times more than 4 standard deviations above each participant's grand mean. The mean percentages of trials thus excluded in the color, lexical and semantic tasks were 0.61%, 0.72%, and 0.84%, respectively.

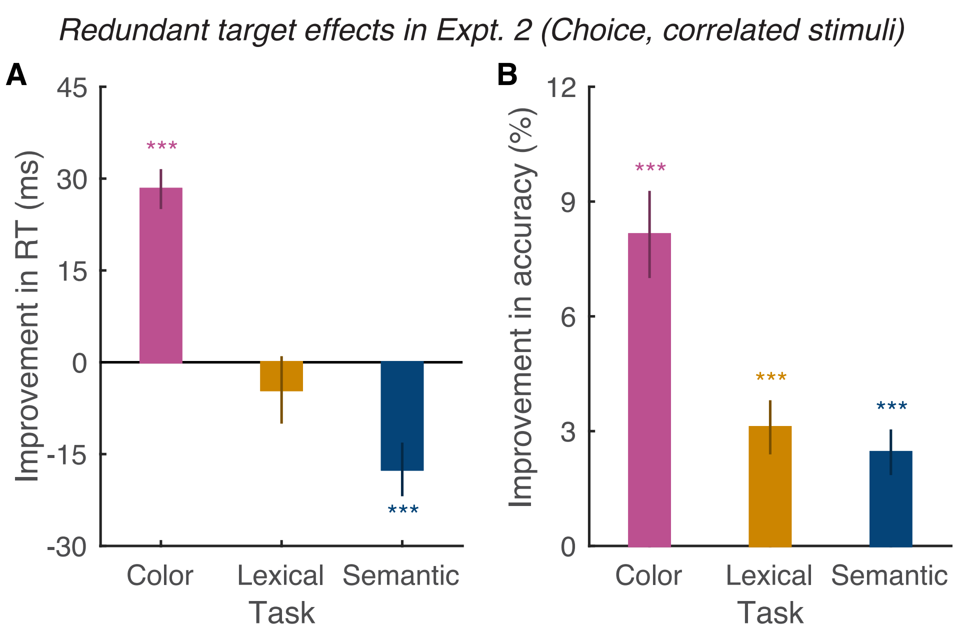

Figure 6: Mean redundant target effects in Experiment 2. Format as in Figure 4.

Results

Response times: **Figure 6A** demonstrates that the redundant target effects on correct response times in Experiment 2 were similar to those in Experiment 1. There was a significant speeding of response times in the color task (by 28 ms on average), no significant effect in the lexical task (-5 ms), and a significant *slowing* in the semantic task (by -18 ms). Statistics on the effect for each task are reported in **Table 4.** Compared to the color task, both the lexical and semantic tasks had significantly different redundant target effects (both F>18, $p<10^{-4}$). The lexical and semantic tasks yielded effects that were not significantly different ($F(1, 15141)=2.61$, $p=0.11$). See the top row of **Figure 7** for mean correct response times in each condition separately.

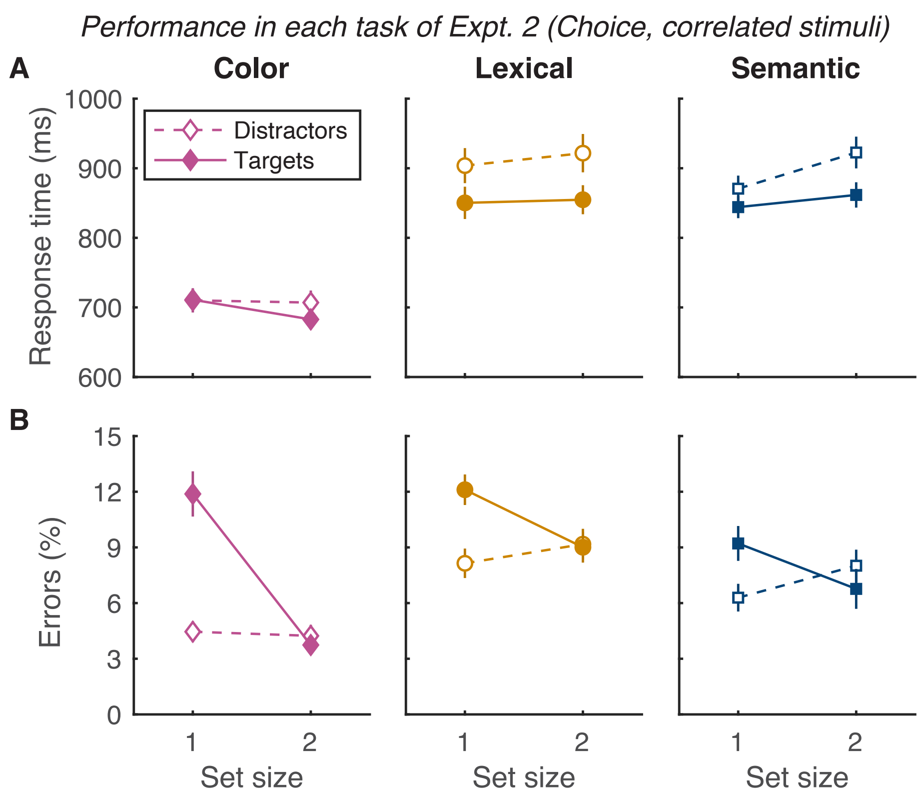

Figure 7. Mean performance in each task of Experiment 2. Format as in Figure 4.

Accuracy: **Figure 6B** plots the mean improvements in accuracy caused by redundant targets, which were significant in all three tasks (as also reported in Table 4).

The mean percent errors in each condition are plotted in the bottom row of **Figure 7**. Compared to the color task, both the lexical and semantic tasks had smaller redundant target effects on accuracy (both F>21, $p<10^{-5}$). The redundant target effects did not differ significantly between the lexical and semantic tasks (F(1, 16689)=0.05, p=0.82).

Task	Effect mean	Effect SEM	95% CI	*t*	*p*	BF
	Correct response times (ms)					
Color	28.29	3.26	[22 35]	8.52	$1.3\text{x}10^{-08}$	$4.1\text{x}10^{6}$
Lexical	-4.51	5.51	[-15 6]	0.80	0.48	0.27
Semantic	-17.50	4.38	[-26 -10]	-3.92	0.001	57.8
	Errors (percent)					
Color	8.14	1.14	[6.04 10.85]	7.02	$1.1\text{x}10^{-6}$	$1.2\text{x}10^{5}$
Lexical	3.10	0.71	[1.71 4.70]	4.30	0.001	142.9
Semantic	2.45	0.60	[1.26 3.55]	4.03	0.001	74.8

Table 4: Statistics on redundant target effects in Experiment 2, formatted as in Table **3**. The degrees of freedom was 28 for the color task and 27 for the others.

Discussion

The results of Experiment 2, which used a choice procedure, were consistent with the results of Experiment 1, which used a go/no-go procedure. The redundant target effects on response times were consistent with parallel processing in the color task and serial processing in the lexical and semantic tasks. Again, the redundant target effect in the semantic task was significantly negative.

<u>Experiment 3: Forced-choice procedure with uncorrelated stimuli</u>

In both experiments reported so far, the words presented simultaneously on trials with set size 2 were always of the same category (both targets or both distractors, although never the same exact words). That is what we mean by a 'correlated' stimulus design. One potential drawback of this design is that the participant might adopt a strategy of always processing just one word, knowing that the other word leads to the same correct decision. (Although they cannot simply pick a *side* of the screen and always ignore stimuli presented on the other side, because half the trials contain just a single word that could be on either side, unpredictably). In the third experiment, we addressed this issue by including trials in which a target is paired with a distractor. Thus, the stimuli are uncorrelated. This should motivate the participant to process both stimuli as well as they can. Uncorrelated stimuli like this are also common in the wider visual search literature.

Methods

Participants**:** Participants in the lexical and semantic tasks were recruited and compensated in the same way as in Experiment 1, via Prolific. Participants in the color task were recruited from the Barnard College Introductory Psychology subject pool, and participated in exchange for course credit. See Table 1 for counts. One participant was excluded for overall accuracy less than 0.6, and another because they pressed the same key on every trial (but overall proportion correct was greater than 0.6 because targets were present on 75% of trials).

Stimuli and procedure: All details were the same as in Experiment 2, except as noted here. The primary difference is that 30% of trials contained mixed pairs of 1 target and 1 distractor. **Table 5** lists the proportions of trials assigned to each type, which were chosen to ensure that the categories (target vs distractor) of the upper and lower stimuli were independent (uncorrelated). Specifically, on two-word trials, the conditional probability of one stimulus being a target given that the other was a target was 0.5. In contrast, this conditional probability was 1.0 in Experiment 1 and 2. Another difference in Experiment 3 was that across the experiment, the probability of a target being present on any given trial was 0.75, rather than 0.5. That was true both among trials with set size 1 and trials with set size 2. To maintain a the same number of two-target trials as in Experiments 1 and 2, we increased the total number of trials in Experiment 3 to 1000. Each participated conducted 10 blocks of 100 trials each.Also, six words were added to the stimulus set for the color and semantic tasks (see the online data repository).

		Bottom word		
		None	*Distractor*	*Target*
Top word	*None*	*N/A*	*0.05*	*0.15*
	Distractor	*0.05*	0.15	0.15
	Target	*0.15*	0.15	0.15

Table 5: The probability of stimulus pairings at the top and bottom locations in Experiment 3. The word at each location was either absent, a distractor, or a target. The green shading highlights conditions when 2 words were present. In this design, the two stimuli were uncorrelated and independent: the conditional probability of one stimulus being a target given that the other was a target was 0.5.

<u>Analysis:</u> We analyzed these data in the same way as Experiment 1 and 2, focusing on the comparison between trials with two targets and trials with a single target presented alone, which provide the best test of our self-terminating models of parallel or serial processing. The mean percentages of trials excluded for sluggish response times (>4 SDs above each participant's mean) in the color, lexical and semantic tasks were 0.48%, 0.56%, and 0.61%, respectively.

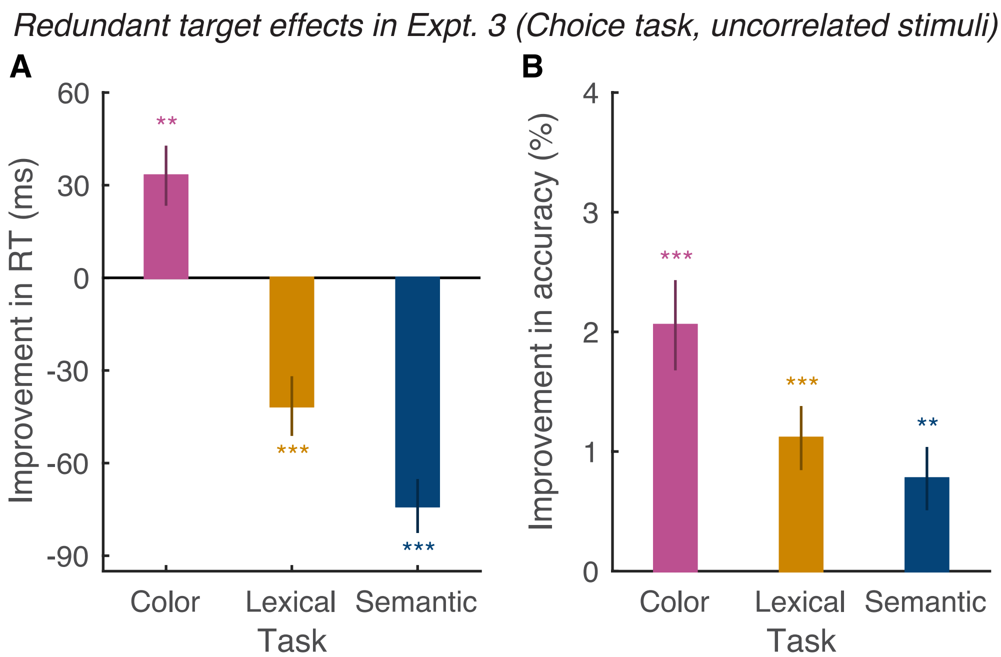

Figure 8: Redundant target effects in Experiment 3. Format as in Figures 4 and 6. These bar plots show the mean *improvement* in correct response time (RT) and accuracy comparing trials with 2 targets to trials with just 1 target and 0 distractors. The mixed-pair trials were not included in this analysis (but are plotted in Figure 9).

Results

Response times: **Figure 8A** shows that the redundant target effects were consistent with the prior two experiments, although somewhat magnified. As detailed in **Table 6**, redundant targets improved responses in the color task (by 33 ms on average), but slowed responses in *both* the lexical task (-42 ms) and the semantic task (-74 ms). All three pairwise comparisons between these effects were significant (color vs. lexical and color vs. semantic: both $F>27$, $p<10^{-6}$; lexical vs semantic: $F(24587)=5.70$, $p=0.022$). See the top row of **Figure 9** for mean correct response times in each condition separately, including the mixed target-distractor trials, which are represented with single lightly-shaded symbols.

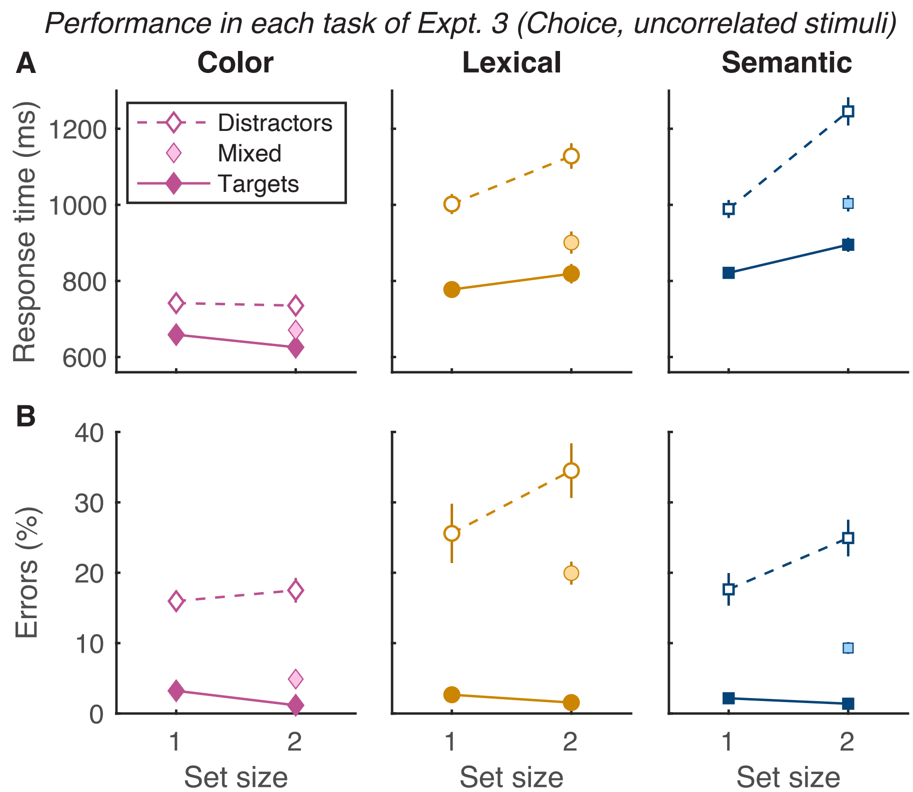

Figure 9. Mean performance in each task of Experiment 3. Format as in Figures 5 and 7, except with the addition of 'mixed' trials that contained 1 target and 1 distractor.

Accuracy: **Figure 8B** plots the positive effects of redundant targets on accuracy, which were significant in all three tasks (see Table 6). None of the pairwise comparisons of these effects across tasks were significant after correcting for multiple comparisons (color vs. lexical: F(25111)=3.45, p=0.095; color vs. semantic: F(25105)=4.10, p=0.077; lexical vs semantic: F(25114)=0.10, p=0.821). The mean percentage errors in each condition are shown in the bottom row of **Figure 9**.

Task	Effect mean	Effect SEM	95% CI	*t*	*p*	BF
	Correct response time (ms)					
Color	33.07	9.73	[21 72]	3.34	0.0032	15.2
Lexical	-41.53	9.67	[-71 -28]	4.22	0.0006	116.5
Semantic	-73.90	8.74	[-93 -58]	8.30	$2x10^{-8}$	$1.9x10^{6}$
	Errors (percent)					
Color	2.06	0.38	[1.34 2.76]	5.35	$5.3x10^{-5}$	$1.8x10^{3}$
Lexical	1.11	0.27	[0.62 1.58]	4.08	0.001	83.5
Semantic	0.77	0.26	[0.26 1.34]	2.87	0.010	5.6

Table 6: Statistics on redundant target effects in Experiment 3, formatted as in Table 3. The degrees of freedom for the t-tests was 27.

Discussion

The results of Experiment 3 were consistent with both prior experiments: there is a positive redundancy target effect on response times only in the color task. One new result in this experiment was that the lexical task, as well as the semantic task, yielded a significantly *negative* redundant target effect (slowing of responses). These results again are consistent with the hypothesis that colors are processed in parallel, but word meanings are processed serially.

We chose to focus our analysis on trials with targets only. Some prior redundant target studies have compared two-target trials to the mixed target-distractor trials, in some cases to test theories of 'coactivation' (C. W. Eriksen et al., 1989; Miller, 1982; Mordkoff & Yantis, 1991). Other experiments with words have compared trials with two targets to trials with one target and a "filler" pseudoword stimuli that was neither a target nor distractor (Shepherdson & Miller, 2014). However, such contrasts do not clearly distinguish our self-terminating serial model from the parallel models. Even the serial model predicts faster responses to two targets than to a single target paired with a non-target, because if the non-target is processed first, search must continue. As shown in Figure 9, mean responses to mixed-pair trials were in fact slower (and less accurate) than responses to trials with two targets. Thus, we have focused on comparing two-target trials (set size 2) to single-target trials (set size 1). The result of that contrast was dramatically different across tasks.

General Discussion

The three experiments reported above consistently demonstrate that there is a positive redundant target effect when the targets are defined by color but not when the targets are defined by lexicality or by semantic category. In all three tasks, the stimuli were written words presented singly or in pairs above and/or below fixation. In the color task, the presence of a redundant target (a word written in colored letters) consistently sped responses compared to trials with a single target. In the semantic task, the presence of a redundant target (a word that refers to a living thing) *slowed* responses in each experiment. In the lexical decision task, the redundant target had no effect in two experiments (with correlated stimuli), and a significantly negative effect in the third experiment (with uncorrelated stimuli).

These data are all consistent with the hypothesis that the low-level features, such as color, of multiple stimuli are processed in parallel, but written words are *recognized* serially. The lexical decision and semantic categorization tasks were two ways to assess word recognition: the lexical decision task requires the participant to judge the familiarity of each letter string, and the semantic categorization task further requires further judgment of the word's meaning.

Relation to previous redundant target studies of word recognition

In contrast to our key result, some prior studies of word recognition have reported positive redundant target effects. However, several of those experiments presented two copies of the same word on redundant target trials (Egeth et al., 1989; Hasbrooke &

Chiarello, 1998; Mohr et al., 1994, 1996; Mullin & Egeth, 1989, Experiment 1-2). The resulting redundant target effects might be explained by facilitation or co-activation at the stage of letter or syllable processing, for example, rather than actual semantic recognition. The only study to report a positive redundant target effect that did not present identical pairs of words was by Shepherdson & Miller (2014). They found that semantic categorization judgments were faster for two targets than for a single target paired with a pronounceable non-word. This could be interpreted as a positive redundant target effect and evidence for parallel processing. However, the result can be explained by the serial model if we assume that on some one-word trials, the participant processes the non-word before they process the target. Therefore, we consider the strict test of our serial model to be the contrast between trials with two targets (which are two different words) and trials with a single target presented alone.

Thus far, all experiments that made this strict test of the serial model with word recognition tasks lead to the same conclusion. They were conducted by us in the present study and by Mullin & Egeth (1989). The experiments in that prior study were like our Experiment 1: they presented words above and/or below fixation and used go/no-go target detection tasks in which targets were never presented with distractors (a correlated stimulus design). In two of their experiments, the words presented together on redundant-target trials were *not* identical (as in ours). In those experiments, they found that both lexical decision and semantic categorization judgments were *slowed* by the presence of a redundant target – but significantly so only in the lexical task. Based on these results, the authors rejected the standard, self-terminating, unlimited-capacity parallel processing model for recognizing two words, as do we. They raised several tentative explanations for how 'interference' between two words might cause the negative redundant target effect. But as we discussed, this negative effect is predicted by the standard serial model that accounts for errors in stimulus classification.

We also go beyond previous studies by showing that the conclusions hold for choice tasks (Experiment 2) and well as go-no/go tasks (Experiment 1), and when the two stimuli are 'uncorrelated', meaning that targets can appear with distractors (Experiment 3). Importantly, we also contrasted the word recognition tasks to a color task. The color task served as a positive control that demonstrated that redundant target effects are possible with the same stimuli presented at the same locations.

Thus, we can reject the standard serial model for the color task, but not for the word recognition tasks (i.e., lexical decision and semantic categorization). In addition, we can reject the standard unlimited-capacity, parallel model for the word recognition tasks, but not for the color tasks. What we cannot do is reject the standard *fixed-capacity*, parallel model for the word recognition tasks. That does not mean that the fixed-capacity model is always viable. It can predict the wide range of redundant target effects, depending on the assumptions built into it and the specific parameters. More work is needed to test those assumptions and parameters.

Relation to the wider literature on serial versus parallel word recognition

Two related questions have fueled many studies of visual word recognition and reading: (1) *Can* multiple words be recognized in parallel? (2) In natural reading, *do* readers process multiple words in parallel? Both questions are heavily debated. The redundant target effects explored in the present article are one way to investigate the first question. The second question arose earlier, however, so we review it next and then return to the first question.

There is ample evidence that when fixating word *n*, readers begin processing word *n+1* before the eyes move (e.g., "parafoveal preview"; Schotter et al., 2012). That might either mean that attention is distributed over multiple words simultaneously, or that before the eyes move from word *n*, processing shifts covertly to word *n+1*, and words are processed serially. Several computational models predict such phenomena assuming either parallel or serial processing of individual words (Engbert et al., 2005; Reichle et al., 2006; Reilly & Radach, 2006; Snell, van Leipsig, et al., 2018). The debate between such models has proven intractable over the years.

One interesting empirical result during a naturalistic reading task is the *transposed word effect*: readers often fail to notice when the order of two words has been reversed (Mirault et al., 2018). That might be evidence that readers process multiple words in parallel with imperfect position coding (Snell & Grainger, 2019b). However, the transposed word effect also occurs when the words in each sentence are presented one at a time, serially, and parallel processing is not possible (Hossain & White, 2023; Huang & Staub, 2022; Liu et al., 2022; Milledge et al., 2023; but see Mirault et al., 2022).

Given the complexity of natural reading and the wide range of models to explain it, other researchers (including the present authors) have turned to more controlled experimental paradigms. The goal is to investigate the fundamental processing capacity limits of visual word recognition – are readers even capable of recognizing two words in parallel, when they are forced to try?

One such paradigm is the unspeeded dual-task paradigm that measures accuracy. It has provided evidence for a serial "bottleneck" in word recognition (White, Boynton, et al., 2019). In these experiments, participants are presented with two words at once. They must either response to one pre-cued word (with focused attention) or respond to both words in sequence (with divided attention). A key difference from the redundant target paradigm is that the two words must be judged independently, rather than integrated to one decision. Also, the primary measure is accuracy rather than response time, and several studies have post-masked the words after an interval calibrated to each individual's performance in the single-task condition. Thus, each participant is given just enough time to process one word, and the question is whether they can process two words with divided attention in that same amount of time. Parallel and serial processing models make predictions for the magnitude of the drop in accuracy in the divided attention condition compared to the focused attention condition.

The results of several dual-task studies have been consistent with the serial model: the observer can recognize only one word per trial and must guess when asked about the other. That has been true for semantic categorization and lexical decision judgments, with words positioned above/below fixation, and to the left/right (White et al., 2018, 2020; White, Palmer, et al., 2019). The cost of dividing attention on accuracy in these experiments rejects the standard unlimited or fixed-capacity parallel models. That measure alone cannot reject a more extreme limited-capacity model. However, another result in these studies is a negative correlation between the two responses made within the same trial. The response to one stimulus was more likely to be correct when the response to the other stimulus was incorrect. This result is consistent with the standard serial model and rejects *all* standard parallel models. To account for the negative correlation, the parallel model would need an ad-hoc addition.

By varying the types of judgements that participants must make about pairs of words, dual-task experiments have shed some light on the source of the processing bottleneck. When the task was to judge the colors of the letters, rather than the meaning of the words, accuracy was consistent with an unlimited-capacity or modestly limited-capacity, parallel model (White et al., 2018, 2020). When the task was to detect vowels within letter strings, or the judge the pronounceability of letter strings, accuracy was again consistent with the standard serial model (Campbell et al., 2024). These data lead to a similar conclusion as the redundant target studies reported in the present article: low-level features are processed in parallel, but linguistic features of words are processed serially. In addition, one neuroimaging study identified a potential neural locus of the serial bottleneck in the left ventral temporal cortex (White, Palmer, et al., 2019).

A related paradigm is called "partially-valid cueing," in which one of two stimulus locations is pre-cued as more likely to be task-relevant. One study using post-masked words found that when participants were asked to judge the semantic category of a word that appeared at the uncued (less attended) location, they performed no better than chance (Johnson et al., 2022). This is again consistent with the standard serial model for word recognition. On the face of it, this result is inconsistent with any parallel model. If processing in parallel, why not acquire some information about the low-probability word? To save the parallel model, one must assume a strategy of processing only one word at a time under some conditions; in other words, the parallel model becomes effectively serial.

Not all studies agree, however (Snell & Grainger, 2019a). Varieties of the "flanker paradigm" (Eriksen & Eriksen, 1974) have demonstrated that judgments of one target word are influenced by the characteristics of nearby, task-irrelevant words (Snell et al., 2017; Snell, Mathôt, et al., 2018; Snell & Grainger, 2018, but see Broadbent & Gathercole, 1990). That is true even when the whole display is flashed for 50 ms and then masked (Snell, 2024). Moreover, there is a "sentence superiority effect": when the words that are flashed along with the target form a sentence, the target is reported more accurately than when the word order is scrambled (Snell & Grainger, 2017; Wen et al., 2019). One interpretation of these results is that the words in each display were all processed simultaneously (Snell & Grainger, 2019b). It is important to note that these experiments differ from the redundant target experiments reported above in at least two key ways: (1) the target word was fixated directly and flankers were arranged horizontally to the left and right; (2) only one word was task-relevant, so the influence of flankers is probably due to automatic processing.

Thus, prior research on the capacity for processing of multiple words has yielded some inconsistent results. The dual-task studies, in which participants explicitly attempt to recognize two unrelated words at once, have so far been consistent with a serial model. The words in those studies have been positioned in the parafovea and thus may not capture the processing capacity of natural reading. The flanker effect studies have been interpreted as evidence for parallel processing of multiple words (arranged horizontally). These flanker-effect studies have not as strictly controlled the time available to process each display, nor tested specific quantitative models of serial processing.

The data reported in the present article add to that prior research in several ways. The redundant target paradigm complements the dual-task paradigm because the words do not have to be post-masked, and the observer needs not make independent judgements about two words simultaneously. Moreover, the effects on response time can be compared to specific quantitative models. Altogether, our results so far are consistent with the dual-task studies in supporting the serial model. It is important to note, though, that we have tested only displays with words placed above and below the fixation point, rather than in more naturalistic arrangements.

Relation to previous theory on response time and accuracy

In this article, we have emphasized developing a general theory of redundant target effects that predicts both response time and accuracy. It is general in the sense that it does not assume any particular stochastic process or response time distribution. Most previous work has followed one of two paths. One is to develop a pure response time theory that ignores errors (e.g., Townsend & Nozawa, 1995). This work is also general in not assuming particular stochastic processes or response time distributions.

The second path is to assume a specific stochastic process such as the diffusion process or the linear ballistic accumulator (both described in the Appendix). For example, Blurton, Greenlee & Gondan (2014) built on the diffusion process to model the redundant target effect. The strength of this path is the integrated treatment of response time and accuracy.

Here we sought to expand the general response time models to incorporate errors. The surprising result was that the standard, self-terminating, serial model with errors showed a *negative* redundant target effect. This is not predicted by the corresponding pure response time model. This work complements other recent effects to generalize pure response time models. In particular, Little et al. (2022) extended part of the theory of systems factorial technology to include errors. They examine the prediction of the double-factorial paradigm to distinguish serial and parallel processes. They showed that the previous analysis of exhaustive search models was general to conditions with errors. However, they did not find a similar general result for self-terminating search models. Instead, they examined two special cases and showed that the analysis for pure response time did generalize to those cases. This is important progress, but it remains to be determined if this method of distinguishing serial and parallel models holds for *all* standard, self-terminating models with errors.

In summary, a critical development is the creation of general theories of both response time and accuracy. We have developed such a theory for the redundant target paradigm.

Conclusion

This study makes two primary contributions: first, we developed a new theory of the redundant target effect, which yielded some new results. By accounting for errors as well as response time, the standard, self-terminating serial model predicts that redundant targets *slow* correct responses, even when they increase accuracy. In contrast, the standard, self-terminating, unlimited-capacity parallel model always predicts positive redundant target effects, even when allowing for errors. We also developed specific examples of standard, fixed-capacity parallel models, which can predict a wide range of redundant target effects. Second, we presented experimental tests of these predictions for judgements of words. When the task required judgment of the letter colors, a positive redundant target effect rejected the standard serial model. When the task required recognizing the words, zero or negative redundant target effects rejected the standard, unlimited-capacity parallel model and was instead consistent with the standard serial model. Thus, the redundant target paradigm shows promise for discriminating parallel and serial models.

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Transparency & Openness statement: We report how we determined our sample size and all data exclusions in the study. All stimuli, data, and analysis code are available at: https://osf.io/a9kqj/. Data were analyzed using MATLAB version 2022A (Mathworks). This study's design and analysis were not pre-registered.

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APPENDIX

This appendix describes three closely matched models of the redundant target effect: one serial and two parallel. These models start from the standard self-terminating search models (e.g., Townsend & Nozawa, 1995) and add an account of accuracy. The main new result is that when there are errors, the standard self-terminating serial model predicts a slower response with two targets compared to one. This contrasts with the corresponding self-terminating serial model without errors that predicts no effect on response time. Another new result concerns the prediction of the standard self-terminating, unlimited-capacity, parallel model with errors. It predicts that the response time for two targets is faster than for one target. This is in accord with the corresponding parallel model without errors, although the effect is reduced with errors. Finally, for the standard fixed-capacity, parallel model there are no general predictions. Thus, among these landmark models, the redundant target paradigm can help distinguish serial and parallel processing.

Task Description

We focus on typical yes-no visual search tasks in which one or two stimuli are presented. A stimulus can either be a target t or a distractor d. When the task has one stimulus, the possible stimuli are t and d. When the task has two stimuli, the possible stimuli are tt, dd and td. Thus, the entire set of possible stimuli is $S = \{t, d, tt, dd, td\}$. The task is to respond "yes" to the presence of any target, and respond "no" to the absence of any target. The notation is summarized in a table at the end of the appendix.

There are three primary response measures of the redundant target task to be predicted that are subscripted by the stimulus condition: probability of a correct response p_s, the mean correct response time $\mu_{s,correct}$, and the mean incorrect response time $\mu_{s,incorrect}$ for $s \in S$. For example, for the single target condition t, these variables are denoted: p_t, $\mu_{t,correct}$, and $\mu_{t,incorrect}$.

Standard Self-terminating Serial Model with Errors

As with typical models of pure response time without errors, our serial model is based upon the selective influence of each stimulus on a separate process. In other words, there is one stimulus-specific component process for each stimulus. Each component process mediates the effect of one stimulus on both response time and accuracy.

For each possible stimulus $s \in \{t, d\}$ and associated component process, define a binary random variable for accuracy by $\boldsymbol{Z}_s$, which has a value of 1 if the decision is correct for that individual stimulus, and a value of 0 if incorrect. In addition, denote the random variables for the component processing time for a correct decision for stimulus s by $\boldsymbol{D}_{s,correct}$, and for an incorrect decision by $\boldsymbol{D}_{s,incorrect}$. We emphasize that these are decisions are about a single stimulus and not about the response made to the set of stimuli.

As with similar models, assume that, besides the component processes, there are other "residual" processes that do not depend on the stimulus and do not affect accuracy, but contribute to the response time. The processing time from these residual processes is denoted by a random variable $\boldsymbol{R}$ and it additively combines with the stimulus-specific component processes to yield the response time. We allow this residual processing time to depend on the specific response regardless of the stimulus. The random variable $\boldsymbol{R}_{yes}$ represents the residual processing time when the response is "yes" indicating the presence of a target, and $\boldsymbol{R}_{no}$ when the response is "no" indicating the absence of a target.

As with standard models of response times without errors, we assume a strong degree of independence, termed *context independence,* between component processes in terms of accuracy and the component processing time. The component accuracy to one stimulus is assumed to be independent of other stimuli in the same stimulus condition. Consider a stimulus condition with two stimuli, denoted s_1 and s_2, with $s_1, s_2 \in \{t, d\}$. Under this assumption, the random variable for component accuracy to stimulus s_1, $\boldsymbol{Z}_{s_1}$, is independent of the random variable for component accuracy to stimulus s_2, $\boldsymbol{Z}_{s_2}$. Furthermore, when $s_1 = s_2$ (either both targets or both distractors), the component accuracies are identically distributed, that is, $\boldsymbol{Z}_{s_1}$ is identically distributed to $\boldsymbol{Z}_{s_2}$.

Similarly, the component processing times are unaffected by the context for a given stimulus. They are independent of each other, and when the stimulus is the same (either both targets or both distractors), are identically distributed. Specifically, for a stimulus condition with two stimuli s_1 and s_2, with $s_1, s_2 \in \{t, d\}$, all pairs of the random variables $\boldsymbol{D}_{s_1,correct}$, $\boldsymbol{D}_{s_2,correct}$, $\boldsymbol{D}_{s_1,incorrect}$, $\boldsymbol{D}_{s_2,incorrect}$, $\boldsymbol{Z}_{s_1}$, and $\boldsymbol{Z}_{s_2}$ are independent, often referred to as stochastic independence. Additionally, the random variables $\boldsymbol{R}_{yes}$ and $\boldsymbol{R}_{no}$ are independent of the other random variables. Moreover, when $s_1 = s_2$, then $\boldsymbol{D}_{s_1,correct}$ is identically distributed to $\boldsymbol{D}_{s_2,correct}$ and $\boldsymbol{D}_{s_1,incorrect}$ is identically distributed to $\boldsymbol{D}_{s_2,incorrect}$.

Context independence, including the accuracy and component processing assumptions, subsumes the more specific independence assumptions of independence from set size (unlimited capacity) and independence from processing order (in the serial model). For parallel models, we separate the unlimited-capacity assumption from context independence to allow consideration of limited capacity.

Even with context independence, the accuracy and component times within a single component process are not constrained and can be dependent. Specifically, for any $s \in \{t, d\}$, there is no restriction between $\boldsymbol{Z}_s$, $\boldsymbol{D}_{s,correct}$, and $\boldsymbol{D}_{s,incorrect}$. This is because correct and incorrect component processing times can differ, and are represented by separate variables. This allows the mean incorrect response time to be slower or faster than the mean correct response time.

Predictions of the Standard Self-terminating Serial Model with Errors

First consider stimulus conditions with either a single target t or two targets tt. Our goal is to describe the component processes of the two-stimulus conditions in terms of the component processes of a single stimulus.

For a **single target condition** t, the predicted probability of a correct response is defined as

$$p_t = P(\boldsymbol{Z}_t = 1). \tag{1}$$

The random variable for the correct response time to a target is,

$$\boldsymbol{T}_{t,correct} = \boldsymbol{D}_{t,correct} + \boldsymbol{R}_{yes}.$$

The expected response time for a correct response to a single target is then the sum of the expected values of that $\boldsymbol{D}_{t,correct}$ and $\boldsymbol{R}_{yes}$,

$$\mu_{t,correct} = \mathrm{E}\left[\boldsymbol{D}_{t,correct}\right] + \mathrm{E}\left[\boldsymbol{R}_{yes}\right]. \tag{2}$$

Relying on stochastic independence, $\boldsymbol{D}_{t,correct}$ and $\boldsymbol{R}_{yes}$ are independent and therefore the variance of the response time of a correct response is the sum of the variances,

$$\sigma^2_{t,correct} = \mathrm{Var}[\boldsymbol{D}_{t,correct}] + \mathrm{Var}[\boldsymbol{R}_{yes}].$$

Similarly, the random variable for the incorrect response time to a target is,

$$\boldsymbol{T}_{t,incorrect} = \boldsymbol{D}_{t,incorrect} + \boldsymbol{R}_{no}$$

with expected value and variance as,

$$\mu_{t,incorrect} = \mathrm{E}[\boldsymbol{D}_{t,incorrect}] + \mathrm{E}[\boldsymbol{R}_{no}]$$

$$\sigma^2_{t,incorrect} = \mathrm{Var}[\boldsymbol{D}_{t,incorrect}] + \mathrm{Var}[\boldsymbol{R}_{no}].$$

For the **two-target condition** tt, consider three mutually exclusive cases that describe the possible processing sequences of this serial model:

Case 1: One stimulus is processed first and is correct, and then processing is terminated (ignoring the other stimulus).

Case 2: One stimulus is processed first and is incorrect, and then the other stimulus is processed correctly.

Case 3: One stimulus is processed first and is incorrect, and then the other stimulus is processed and is also incorrect.

The probabilities and response times follow by case.

Case 1: The probability that Case 1 occurs is equal to the probability that a single stimulus is processed correctly, because of context independence,

$$p_{tt,Case1} = P(\boldsymbol{Z}_t = 1) = p_t.$$

The correct response time for Case 1 is the same as the response time for a correct response to a single target, also because of context independence,

$$\boldsymbol{T}_{tt,Case1} = \boldsymbol{D}_{t,correct} + \boldsymbol{R}_{yes}.$$

The expected correct response time for Case 1 is,

$$\mu_{tt,Case1} = E[\boldsymbol{D}_{t,correct}] + E[\boldsymbol{R}_{yes}].$$

Due to stochastic independence, the variance of the correct response time for Case 1 is,

$$\sigma^2_{tt,Case1} = \mathrm{Var}[\boldsymbol{D}_{t,correct}] + \mathrm{Var}[\boldsymbol{R}_{yes}].$$

Case 2: The probability that Case 2 occurs is equal to the probability that one stimulus is processed first and is incorrect, and that the other stimulus is processed correctly. Let t_{first} denote the target processed first, and t_{second} denote the target processed second. By context independence and stochastic independence, the probability of Case 2 can be expressed in terms of single stimulus probabilities,

$$p_{tt,Case2} = P\left(\boldsymbol{Z}_{t_{first}} = 0 \text{ and } \boldsymbol{Z}_{t_{second}} = 1\right) = (1 - p_t)p_t.$$

The correct response time for Case 2, also relying on context independence, is the processing time for an incorrect response to one stimulus plus the processing time for a correct response to the other stimulus,

$$\boldsymbol{T}_{tt,Case2} = \boldsymbol{D}_{t,incorrect} + \boldsymbol{D}_{t,correct} + \boldsymbol{R}_{yes}.$$

The expected correct response time for Case 2 is,

$$\mu_{tt,Case2} = E[\boldsymbol{D}_{t,incorrect}] + E[\boldsymbol{D}_{t,correct}] + E[\boldsymbol{R}_{yes}].$$

The variance of the correct response time for Case 2, relying on stochastic independence, is,

$$\sigma^2_{tt,Case2} = \mathrm{Var}\big[\boldsymbol{D}_{t,incorrect}\big] + \mathrm{Var}\big[\boldsymbol{D}_{t,correct}\big] + \mathrm{Var}\big[\boldsymbol{R}_{yes}\big].$$

Case 3: The probability that Case 3 occurs in the serial model is equal to the probability that both stimuli are processed incorrectly. By context independence and stochastic independence,

$$p_{tt,Case3} = P(\boldsymbol{Z}_t = 0 \text{ and } \boldsymbol{Z}_t = 0) = (1 - p_t)^2.$$

The incorrect response time for Case 3 in the serial model is the processing time for an incorrect response to one stimulus plus the processing time for an incorrect response to the other stimulus,

$$\boldsymbol{T}_{tt,Case3} = 2\,\boldsymbol{D}_{t,incorrect} + \boldsymbol{R}_{no}.$$

The expected incorrect response time for Case 3 is,

$$\mu_{tt,Case3} = 2E[\boldsymbol{D}_{t,incorrect}] + E[\boldsymbol{R}_{no}].$$

The variance of the incorrect response time for Case 3 is,

$$\sigma^2_{tt,Case3} = 2\,\mathrm{Var}\big[\boldsymbol{D}_{t,incorrect}\big] + \mathrm{Var}[\boldsymbol{R}_{no}].$$

In the two-target condition, a correct response is achieved in Case 1 *and* in Case 2, resulting in a mixture distribution (see Chatfield & Theobald, 1973). The probability of a correct response is the probability of Case 1 plus the probability of Case 2, because they are mutually exclusive,

$$\begin{aligned} p_{tt} &= p_{tt,Case1} + p_{tt,Case2} \\ &= p_t + (1 - p_t)p_t \\ &= 2\,p_t - p_t^2. \end{aligned} \tag{3}$$

The expected response time for a correct response to a two-target condition, $\mu_{tt,correct}$, is the weighted average of the expected response times for Case 1 and Case 2. The weight for Case 1 is the proportion of correct responses for Case 1, i.e., the ratio of the probability of Case 1 to the probability of Case 1 or Case 2,

$$w_{Case1} = \frac{p_{tt,Case1}}{\left(p_{tt,Case1} + p_{tt,Case2}\right)} = \frac{p_t}{(2p_t - p_t^2)} = \frac{1}{(2 - p_t)}.$$

Similarly,

$$w_{Case2} = \frac{p_{tt,Case2}}{\left(p_{tt,Case1} + p_{tt,Case2}\right)} = \frac{p_t(1 - p_t)}{(2p_t - p_t^2)} = \frac{(1 - p_t)}{(2 - p_t)}.$$

Using these weights, the expected correct response time is,

$$\begin{aligned} \mu_{tt,correct} &= w_{Case1}E[\boldsymbol{T}_{tt,Case1}] + w_{Case2}E[\boldsymbol{T}_{tt,Case2}] \\ &= w_{Case1}\mu_{tt,Case1} + w_{Case2}\,\mu_{tt,Case2} \\ &= \left(\frac{1}{2 - p_t}\right)E[\boldsymbol{D}_{t,correct} + \boldsymbol{R}_{yes}] + \left(\frac{1 - p_t}{2 - p_t}\right)E[\boldsymbol{D}_{t,incorrect} + \boldsymbol{D}_{t,correct} + \boldsymbol{R}_{yes}] \\ &= E[\boldsymbol{D}_{t,correct}] + \left(\tfrac{1-p_t}{2-p_t}\right)E[\boldsymbol{D}_{t,incorrect}] + E[\boldsymbol{R}_{yes}]. \end{aligned} \tag{4}$$

The variance of the correct response time for a mixture is the sum of two parts: the first part is the weighted averages of the variances of each case, and the second part is the variance due to the differences in the means of the cases. This is called the law of total variance (Chatman and Theobald, 1973), and yields,

$$\sigma^2_{tt,correct} = (w_{Case1}\sigma^2_{tt,Case1} + w_{Case2}\,\sigma^2_{tt,Case2}) + \left(w_{Case1}\mu^2_{tt,Case1} + w_{Case2}\,\mu^2_{tt,Case2} - \mu^2_{tt,correct}\right).$$

To obtain the expected incorrect response time, one can use the results for Case 3 because that is the only case that results in an incorrect response,

$$\mu_{tt,incorrect} = E[\boldsymbol{T}_{tt,Case3}] = 2E[\boldsymbol{D}_{t,incorrect}] + E[\boldsymbol{R}_{no}].$$

The variance of the incorrect response time is the same as that for Case 3,

$$\sigma^2_{tt,incorrect} = 2\,\mathrm{Var}\left[\boldsymbol{D}_{t,incorrect}\right] + \mathrm{Var}[\boldsymbol{R}_{no}].$$

Our main focus is on the difference between the correct response time for one target and the correct response time for two targets. The difference is constructed so that a faster response time to two targets results in a positive difference. Using Equations (2) and (4), the difference is,

$$\begin{aligned}\mu_{t,correct} - \mu_{tt,correct} &= \left(E\left[\boldsymbol{D}_{t,correct}\right] + E\left[\boldsymbol{R}_{yes}\right]\right) \\ &\quad - \left(E[\boldsymbol{D}_{t,correct}] + \left(\frac{1-p_t}{2-p_t}\right)E[\boldsymbol{D}_{t,incorrect}] + E[\boldsymbol{R}_{yes}]\right) \\ &= -\left(\frac{1-p_t}{2-p_t}\right)E[\boldsymbol{D}_{t,incorrect}]. \qquad (5)\end{aligned}$$

This difference is less than or equal to zero. When there are no errors ($p_t = 1$), the correct response times are equal and difference equals zero. Thus, the serial model predicts, in the presence of errors, that a correct response time for two targets is slower than the correct response time for one target.

Next consider the difference between the probability of a correct response for two targets and the probability of a correct response for one target. The difference is constructed so that an increase in accuracy for two targets results in a positive difference. Using Equations (1) and (3), the difference is,

$$p_{tt} - p_t = (2\,p_t - p_t^2) - p_t = p_t - p_t^2 \qquad (6)$$

which is greater than or equal to zero. Thus, redundant targets improve accuracy.

For completeness, the other predictions of this serial model are given next.

For a **single distractor condition** d, by definition,

$$p_d = P(\boldsymbol{Z}_d = 1),$$

$$\boldsymbol{T}_{d,correct} = \boldsymbol{D}_{d,correct} + \boldsymbol{R}_{no}, \text{ and}$$

$$\boldsymbol{T}_{d,incorrect} = \boldsymbol{D}_{d,incorrect} + \boldsymbol{R}_{yes}.$$

The expected values and variances are,

$$\mu_{d,correct} = \mathrm{E}\big[\boldsymbol{D}_{d,correct}\big] + \mathrm{E}[\boldsymbol{R}_{no}],$$

$$\sigma^2_{d,correct} = \mathrm{Var}\big[\boldsymbol{D}_{d,correct}\big] + \mathrm{Var}[\boldsymbol{R}_{no}],$$

$$\mu_{d,incorrect} = \mathrm{E}\big[\boldsymbol{D}_{d,incorrect}\big] + \mathrm{E}\big[\boldsymbol{R}_{yes}\big], \text{ and}$$

$$\sigma^2_{d,incorrect} = \mathrm{Var}[\boldsymbol{D}_{d,incorrect}] + \mathrm{Var}[\boldsymbol{R}_{yes}].$$

For the **two-distractor condition** dd, there are three cases. The first case is when both distractors are processed correctly,

$$p_{dd,Case1} = P(\boldsymbol{Z}_d = 1 \; and \; \boldsymbol{Z}_d = 1) = p_d^2, \text{ and}$$

$$\boldsymbol{T}_{dd,Case1} = 2\boldsymbol{D}_{d,correct} + \boldsymbol{R}_{no},$$

with

$$\mu_{dd,Case1} = 2\,\mathrm{E}\big[\boldsymbol{D}_{d,correct}\big] + \mathrm{E}[\boldsymbol{R}_{no}],$$

$$\sigma^2_{dd,Case1} = 2\,\mathrm{Var}\big[\boldsymbol{D}_{d,correct}\big] + \mathrm{Var}[\boldsymbol{R}_{no}].$$

The second case is when one distractor is processed incorrectly, which terminates processing,

$$p_{dd,Case2} = P(\boldsymbol{Z}_d = 0) = 1 - p_d, \text{ and}$$

$$\boldsymbol{T}_{dd,Case2} = \boldsymbol{D}_{d,incorrect} + \boldsymbol{R}_{yes},$$

with

$$\mu_{dd,Case2} = \mathrm{E}\left[\boldsymbol{D}_{d,incorrect}\right] + \mathrm{E}\left[\boldsymbol{R}_{yes}\right],$$

$$\sigma^2_{dd,Case2} = \mathrm{Var}\left[\boldsymbol{D}_{d,incorrect}\right] + \mathrm{Var}\left[\boldsymbol{R}_{yes}\right].$$

The third case is when one distractor is processed correctly but the second distractor is processed incorrectly,

$$p_{dd,Case3} = P(\boldsymbol{Z}_d = 1 \text{ } and \text{ } \boldsymbol{Z}_d = 0) = p_d(1 - p_d), \text{ and}$$

$$\boldsymbol{T}_{dd,Case3} = \boldsymbol{D}_{d,correct} + \boldsymbol{D}_{d,incorrect} + \boldsymbol{R}_{yes},$$

with

$$\mu_{dd,Case3} = \mathrm{E}\left[\boldsymbol{D}_{d,correct}\right] + \mathrm{E}\left[\boldsymbol{D}_{d,incorrect}\right] + \mathrm{E}\left[\boldsymbol{R}_{yes}\right],$$

$$\sigma^2_{dd,Case3} = \mathrm{Var}\left[\boldsymbol{D}_{d,correct}\right] + \mathrm{Var}\left[\boldsymbol{D}_{d,incorrect}\right] + \mathrm{Var}\left[\boldsymbol{R}_{yes}\right].$$

Only the first case yields a correct response, thus

$$p_{dd} = p_{dd,Case1} = p_d^2, \text{ and}$$

$$\boldsymbol{T}_{dd,correct} = 2\boldsymbol{D}_{d,correct} + \boldsymbol{R}_{no}.$$

The expected value and variance for a correct response is

$$\mu_{dd,correct} = 2\ \mathrm{E}\left[\boldsymbol{D}_{d,correct}\right] + \mathrm{E}[\boldsymbol{R}_{no}],$$

$$\sigma^2_{dd,correct} = 2\ \mathrm{Var}\left[\boldsymbol{D}_{d,correct}\right] + \mathrm{Var}[\boldsymbol{R}_{no}].$$

The other two cases yield incorrect responses, yielding a mixture distribution. The weight for Case 2 is the fraction of incorrect responses due to Case 2 relative to all incorrect responses,

$$w_{Case2} = \frac{(1-p_d)}{\big((1-p_d)+p_d(1-p_d)\big)} = \frac{1}{1+p_d}.$$

Similarly, the fraction of incorrect responses due to Case 3 relative to all incorrect responses is,

$$w_{Case3} = \frac{p_d(1-p_d)}{\big((1-p_d)+p_d(1-p_d)\big)} = \frac{p_d}{1+p_d}.$$

The weighted combination of Cases 2 and 3 yields the expected incorrect response time,

$$\begin{aligned}\mu_{dd,incorrect} &= w_{\mathrm{Case2}}E[\boldsymbol{T}_{dd,Case2}] + w_{\mathrm{Case3}}E[\boldsymbol{T}_{dd,Case3}]\\ &= \left(\frac{1}{1+p_d}\right)\big(E[\boldsymbol{D}_{d,incorrect}] + E[\boldsymbol{R}_{yes}]\big)\\ &\qquad + \left(\frac{p_d}{1+p_d}\right)\big(E[\boldsymbol{D}_{d,correct}] + E[\boldsymbol{D}_{d,incorrect}] + E[\boldsymbol{R}_{yes}]\big)\\ &= \mathrm{E}[\boldsymbol{D}_{d,incorrect}] + \left(\frac{p_d}{1+p_d}\right)E[\boldsymbol{D}_{d,correct}] + E[\boldsymbol{R}_{yes}].\end{aligned}$$

The variance of the incorrect response time for the mixture of Cases 2 and 3 is,

$$\begin{aligned}\sigma^2_{dd,incorrect} &= (w_{Case2}\sigma^2_{dd,Case2} + w_{Case3}\,\sigma^2_{dd,Case3})\\ &\qquad + \big(w_{Case2}\mu^2_{dd,Case2} + w_{Case3}\,\mu^2_{dd,Case3} - \mu^2_{dd,incorrect}\big).\end{aligned}$$

For the **one target and one distractor condition** td, there are six mutually exclusive cases. The cases are distinguished by whether the target is processed first (Cases 1, 2, and 3), or the distractor is processed first (Cases 4, 5, and 6). Which stimuli is processed first is considered to be random (probability of 0.5). The first case is that the target is processed first and is correct,

$$p_{td,Case1} = p_t,$$

$$\boldsymbol{T}_{td,Case1} = \boldsymbol{D}_{t,correct} + \boldsymbol{R}_{yes},$$

with

$$\mu_{td,Case1} = \mathrm{E}\big[\boldsymbol{D}_{t,correct}\big] + \mathrm{E}\big[\boldsymbol{R}_{yes}\big],$$

$$\sigma^2_{td,Case1} = \mathrm{Var}\big[\boldsymbol{D}_{t,correct}\big] + \mathrm{Var}\big[\boldsymbol{R}_{yes}\big].$$

The second case is that the target is processed first incorrectly and then the distractor is processed correctly,

$$p_{td,Case2} = (1 - p_t)p_d$$

$$\boldsymbol{T}_{td,Case2} = \boldsymbol{D}_{t,incorrect} + \boldsymbol{D}_{d,correct} + \boldsymbol{R}_{no},$$

with

$$\mu_{td,Case2} = \mathrm{E}\left[\boldsymbol{D}_{t,incorrect}\right] + \mathrm{E}\left[\boldsymbol{D}_{d,correct}\right] + \mathrm{E}[\boldsymbol{R}_{no}],$$

$$\sigma^2_{td,Case2} = \mathrm{Var}\left[\boldsymbol{D}_{t,incorrect}\right] + \mathrm{Var}\left[\boldsymbol{D}_{d,correct}\right] + \mathrm{Var}[\boldsymbol{R}_{no}].$$

The third case is that the target is processed first incorrectly and then the distractor is processed incorrectly,

$$p_{td,Case3} = (1 - p_t)(1 - p_d),$$

$$\boldsymbol{T}_{td,Case3} = \boldsymbol{D}_{t,incorrect} + \boldsymbol{D}_{d,incorrect} + \boldsymbol{R}_{yes},$$

with

$$\mu_{td,Case3} = \mathrm{E}\left[\boldsymbol{D}_{t,incorrect}\right] + \mathrm{E}\left[\boldsymbol{D}_{d,incorrect}\right] + \mathrm{E}\left[\boldsymbol{R}_{yes}\right],$$

$$\sigma^2_{td,Case3} = \mathrm{Var}\left[\boldsymbol{D}_{t,incorrect}\right] + \mathrm{Var}\left[\boldsymbol{D}_{d,incorrect}\right] + \mathrm{Var}\left[\boldsymbol{R}_{yes}\right].$$

The fourth case is that the distractor is processed first incorrectly, which terminates processing,

$$p_{td,Case4} = (1 - p_d),$$

$$\boldsymbol{T}_{td,Case4} = \boldsymbol{D}_{d,incorrect} + \boldsymbol{R}_{yes},$$

with

$$\mu_{td,Case4} = \mathrm{E}\left[\boldsymbol{D}_{d,incorrect}\right] + \mathrm{E}\left[\boldsymbol{R}_{yes}\right],$$

$$\sigma^2_{td,Case4} = \mathrm{Var}\left[\boldsymbol{D}_{d,incorrect}\right] + \mathrm{Var}\left[\boldsymbol{R}_{yes}\right].$$

The fifth case is that the distractor is processed first correctly and then the target is processed correctly,

$$p_{td,Case5} = p_d p_t,$$

$$\boldsymbol{T}_{td,Case5} = \boldsymbol{D}_{d,correct} + \boldsymbol{D}_{t,correct} + \boldsymbol{R}_{yes},$$

with

$$\mu_{td,Case5} = \mathrm{E}\left[\boldsymbol{D}_{d,correct}\right] + \mathrm{E}\left[\boldsymbol{D}_{t,correct}\right] + \mathrm{E}\left[\boldsymbol{R}_{yes}\right],$$

$$\sigma^2_{td,Case5} = \mathrm{Var}\left[\boldsymbol{D}_{d,correct}\right] + \mathrm{Var}\left[\boldsymbol{D}_{t,correct}\right] + \mathrm{Var}\left[\boldsymbol{R}_{yes}\right].$$

The sixth case is that the distractor is processed first correctly and then the target is processed incorrectly,

$$p_{td,Case6} = p_d(1 - p_t),$$

$$\boldsymbol{T}_{td,Case6} = \boldsymbol{D}_{d,correct} + \boldsymbol{D}_{t,incorrect} + \boldsymbol{R}_{no},$$

with

$$\mu_{td,Case6} = \mathrm{E}\left[\boldsymbol{D}_{d,correct}\right] + \mathrm{E}\left[\boldsymbol{D}_{t,incorrect}\right] + \mathrm{E}[\boldsymbol{R}_{no}],$$

$$\sigma^2_{td,Case6} = \mathrm{Var}\left[\boldsymbol{D}_{d,correct}\right] + \mathrm{Var}\left[\boldsymbol{D}_{t,incorrect}\right] + \mathrm{Var}[\boldsymbol{R}_{no}].$$

The probability of a correct response is achieved through the mutually exclusive cases 1, 3, 4 and 5. Specifically, it is the probability that the target is processed first (0.5) and results in Case 1 or Case 3, plus the probability that the distractor is processed first (0.5) and results in Case 4 or Case 5,

$$\begin{aligned} p_{td} &= 0.5(p_t + (1 - p_t)(1 - p_d)) + 0.5((1 - p_d) + p_t p_d) \\ &= 1 - p_d + p_t p_d. \end{aligned}$$

For the four cases that contribute to a correct response, the weights are

$$w_1 = \frac{0.5\, p_{td,Case1}}{p_{td}} = \frac{0.5\, p_t}{(1 - p_d + p_t p_d)},$$

$$w_3 = \frac{0.5\, p_{td,Case3}}{p_{td}} = \frac{0.5\,(1 - p_t)(1 - p_d)}{(1 - p_d + p_t p_d)},$$

$$w_4 = \frac{0.5\, p_{td,Case4}}{p_{td}} = \frac{0.5\,(1-p_d)}{(1-p_d+p_tp_d)},$$

$$w_5 = \frac{0.5\, p_{td,Case5}}{p_{td}} = \frac{0.5\, p_tp_d}{(1-p_d+p_tp_d)}.$$

Using these weights, the expected correct response time is,

$$\mu_{td,correct} = w_1E\left[\boldsymbol{T}_{td,Case1}\right] + w_3E\left[\boldsymbol{T}_{td,Case3}\right] + w_4E\left[\boldsymbol{T}_{td,Case4}\right] + w_5E\left[\boldsymbol{T}_{td,Case5}\right]$$

$$= \frac{0.5p_t}{(1-p_d+p_tp_d)}\left(E[\boldsymbol{D}_{t,correct}] + E[\boldsymbol{R}_{yes}]\right)$$

$$+ \frac{0.5(1-p_t)(1-p_d)}{(1-p_d+p_tp_d)}\left(E[\boldsymbol{D}_{t,incorrect}] + E[\boldsymbol{D}_{d,incorrect}] + E[\boldsymbol{R}_{yes}]\right)$$

$$+ \frac{0.5(1-p_d)}{(1-p_d+p_tp_d)}\left(E[\boldsymbol{D}_{t,incorrect}] + E[\boldsymbol{R}_{yes}]\right)$$

$$+ \frac{0.5p_tp_d}{(1-p_d+p_tp_d)}\left(E[\boldsymbol{D}_{d,correct}] + E[\boldsymbol{D}_{t,correct}] + E[\boldsymbol{R}_{yes}]\right)$$

$$= \frac{0.5p_t(1+p_d)}{(1-p_d+p_tp_d)}E\left[\boldsymbol{D}_{t,correct}\right] + \frac{0.5(1-p_t)(1-p_d)}{(1-p_d+p_tp_d)}E\left[\boldsymbol{D}_{t,incorrect}\right]$$

$$+ \frac{0.5p_tp_d}{(1-p_d+p_tp_d)}E[\boldsymbol{D}_{d,correct}]$$

$$+ \frac{0.5(1-p_d)+(1-p_t)(1-p_d)}{(1-p_d+p_tp_d)}E[\boldsymbol{D}_{d,incorrect}] + E[\boldsymbol{R}_{yes}].$$

The variance of the correct response time as a mixture of Cases 1, 3, 4, and 5 is,

$$\sigma^2_{td,correct} = (w_1\sigma^2_{td,Case1} + w_3\sigma^2_{td,Case3} + w_4\sigma^2_{td,Case4} + w_5\sigma^2_{td,Case5})$$

$$+ \left(w_1\mu^2_{td,Case1} + w_3\mu^2_{td,Case3} + w_4\mu^2_{td,Case4} + w_5\mu^2_{td,Case5} - \mu^2_{td,correct}\right).$$

Similarly, the expected incorrect response time is due to Cases 2 and 6. The weights are

$$w_2 = \frac{0.5\, p_{td,Case2}}{p_{td}} = \frac{0.5\,(1-p_t)p_d}{0.5\,(1-p_t)p_d + 0.5\,p_d(1-p_t)} = 0.5,$$

$$w_6 = \frac{0.5\, p_{td,Case6}}{p_{td}} = \frac{0.5\,p_d(1-p_t)}{0.5\,(1-p_t)p_d + 0.5\,p_d(1-p_t)} = 0.5.$$

Using these weights, the expected incorrect response time is,

$$\mu_{td,incorrect} = w_2 E[\boldsymbol{T}_{td,Case2}] + w_6 E[\boldsymbol{T}_{td,Case6}]$$

$$= 0.5\big(E[\boldsymbol{D}_{t,incorrect}] + E[\boldsymbol{D}_{d,correct}] + E[\boldsymbol{R}_{no}]\big)$$

$$+ 0.5\big(E[\boldsymbol{D}_{d,correct}] + E[\boldsymbol{D}_{t,incorrect}] + E[\boldsymbol{R}_{no}]\big)$$

$$= E\big[\boldsymbol{D}_{t,incorrect}\big] + E\big[\boldsymbol{D}_{d,correct}\big] + E[\boldsymbol{R}_{no}].$$

The variance of the incorrect response time as a mixture of Cases 2 and 6 is,

$$\sigma^2_{td,incorrect} = (w_2\sigma^2_{td,Case2} + w_6\sigma^2_{td,Case6}) + \big(w_2\mu^2_{td,Case2} + w_6\mu^2_{td,Case6} - \mu^2_{td,incorrect}\big).$$

Standard Self-terminating, Unlimited-capacity, Parallel Model with Errors

We generalize the standard self-terminating, unlimited-capacity, parallel model without errors to include errors, and ask whether the introduction of errors changes the prediction. In the literature, the parallel model without errors predicts that the correct response time for two targets is faster than for one target. We examine the prediction when including errors in the generalized parallel model. The task description discussed in the first section is the same, and the notation is the same as introduced for the Standard Self-terminating Serial Model with Errors. The difference from the serial model is that, instead of component processes being executed sequentially, in the parallel model the component processes are executed in parallel._As with the serial model, context independence is assumed with a strong degree of independence between component processes in terms of accuracy and component processing time.

Predictions of the Standard Self-terminating, Unlimited-capacity, Parallel Model with Errors

Consider stimulus conditions with either a single target t or two targets tt. As before, our goal is to describe the processes of the two-stimulus conditions in terms of the single-stimulus component processes. For a single target, the parallel model has the same definition and corresponding prediction as the serial model.

For a **single target condition** t, the predicted probability of a correct response is

$$p_t = P(\boldsymbol{Z}_t = 1). \tag{7}$$

The random variable for the correct response time to a target is,

$$\boldsymbol{T}_{t,correct} = \boldsymbol{D}_{t,correct} + \boldsymbol{R}_{yes}.$$

The expected response time for a correct response to a single target is then the sum of the expected values of that $\boldsymbol{D}_{t,correct}$ and $\boldsymbol{R}_{yes}$,

$$\mu_{t,correct} = \mathrm{E}\left[\boldsymbol{D}_{t,correct}\right] + \mathrm{E}\left[\boldsymbol{R}_{yes}\right]. \tag{8}$$

Similarly, the random variable for the incorrect response time to a target is,

$$\boldsymbol{T}_{t,incorrect} = \boldsymbol{D}_{t,incorrect} + \boldsymbol{R}_{no}$$

with expected value as,

$$\mu_{t,incorrect} = \mathrm{E}\left[\boldsymbol{D}_{t,incorrect}\right] + \mathrm{E}[\boldsymbol{R}_{no}].$$

For the **two-target condition** tt, consider three mutually exclusive cases that describe the possible processing sequences of the parallel model:

Case 1: Each stimulus is processed correctly in parallel. The response is reported as soon as a target is detected whichever process is completed first, and then processing is terminated, even though the other stimulus is partially processed.

Case 2: One stimulus is processed correctly, and the other stimulus is processed incorrectly. The response is reported when the stimulus that is processed correctly completes processing, regardless of whether the other stimulus has completed processing or is partially processed.

Case 3: Each stimulus is processed incorrectly. The response is reported after both stimuli have been processed.

The probabilities and response times follow by case.

Case 1: The probability that Case 1 occurs is equal to the probability that both stimuli are processed correctly. Using the independence of component accuracy to one stimulus (t_1) to the other (t_2), the probability that Case 1 occurs is,

$$p_{tt,Case1} = P\left(\boldsymbol{Z}_{t_1} = 1, \boldsymbol{Z}_{t_2} = 1\right) = p_t^2.$$

The correct response time for Case 1 is the processing time for the stimulus that was completed first plus the residual time, also because of context independence,

$$\boldsymbol{T}_{tt,Case1} = \min\{\boldsymbol{D}_{t_1,correct}, \boldsymbol{D}_{t_2,correct}\} + \boldsymbol{R}_{yes}.$$

The expected correct response time for Case 1 is,

$$\mu_{tt,Case1} = E[\min\{\boldsymbol{D}_{t_1,correct}, \boldsymbol{D}_{t_2,correct}\}] + E[\boldsymbol{R}_{yes}].$$

Case 2: The probability that Case 2 occurs is equal to the probability that one stimulus is processed correctly and that the other stimulus is processed incorrectly. By context independence and stochastic independence, the probability that Case 2 occurs can be expressed in terms of single stimulus probabilities,

$$p_{tt,Case2} = P\left(\boldsymbol{Z}_{t_1} = 0 \text{ and } \boldsymbol{Z}_{t_2} = 1, \text{ or } \boldsymbol{Z}_{t_1} = 1 \text{ and } \boldsymbol{Z}_{t_2} = 0\right) = 2(1 - p_t)p_t.$$

The correct response time for Case 2 is the time that one stimulus is processed correctly, regardless of whether the other stimulus has completed processing or is partially processed. The response has to wait until the correct response has completed processing, no matter whether the incorrect component processing time is greater or less than the correct component processing time. Again, by context independence and stochastic independence, the correct response time for Case 2 is,

$$\boldsymbol{T}_{tt,Case2} = \boldsymbol{D}_{t,correct} + \boldsymbol{R}_{yes}.$$

The expected correct response time for Case 2 is,

$$\mu_{tt,Case2} = E[\boldsymbol{D}_{t,correct}] + E[\boldsymbol{R}_{yes}].$$

Case 3: The probability that Case 3 occurs in the parallel model is equal to the probability that both stimuli are processed incorrectly. By context independence and stochastic independence,

$$p_{tt,Case3} = P\left(\boldsymbol{Z}_{t_1} = 0 \text{ and } \boldsymbol{Z}_{t_2} = 0\right) = (1 - p_t)^2.$$

The incorrect response time for Case 3 in the parallel model is the longest processing time for an incorrect response plus the residual time,

$$\boldsymbol{T}_{tt,Case3} = \max\{\boldsymbol{D}_{t_1,incorrect}, \boldsymbol{D}_{t_2,incorrect}\} + \boldsymbol{R}_{no}.$$

The expected incorrect response time for Case 3 is,

$$\mu_{tt,Case3} = E[\max\{\boldsymbol{D}_{t_1,incorrect}, \boldsymbol{D}_{t_2,incorrect}\}] + E[\boldsymbol{R}_{no}].$$

In the two-target condition, a correct response is achieved in Case 1 *and* in Case 2, resulting in a mixture distribution. The probability of a correct response is the probability of Case 1 plus the probability of Case 2, because they are mutually exclusive,

$$\begin{aligned} p_{tt} &= p_{tt,Case1} + p_{tt,Case2} \\ &= p_t p_t + 2(1 - p_t)p_t \\ &= 2\,p_t - p_t^2. \end{aligned} \tag{9}$$

The expected response time for a correct response to a two-target condition, $\mu_{tt,correct}$, is the weighted average of the expected response times for Case 1 and Case 2. The weight for Case 1 is the proportion of correct responses for Case 1, i.e., the ratio of the probability of Case 1 to the probability of Case 1 or Case 2,

$$w_{Case1} = \frac{p_{tt,Case1}}{\left(p_{tt,Case1} + p_{tt,Case2}\right)} = \frac{p_t p_t}{(2p_t - p_t^2)} = \frac{p_t}{(2 - p_t)}.$$

Similarly,

$$w_{Case2} = \frac{p_{tt,Case2}}{\left(p_{tt,Case1} + p_{tt,Case2}\right)} = \frac{p_t(1 - p_t)}{(2p_t - p_t^2)} = \frac{(1 - p_t)}{(2 - p_t)}.$$

Using these weights, the expected correct response time is,

$$\mu_{tt,correct} = w_{Case1}E[\boldsymbol{T}_{tt,Case1}] + w_{Case2}E[\boldsymbol{T}_{tt,Case2}]$$

$$= w_{Case1}\mu_{tt,Case1} + w_{Case2}\,\mu_{tt,Case2}$$

$$= \left(\frac{p_t}{2-p_t}\right)\left(E\left[\min\left\{\boldsymbol{D}_{t_1,correct}, \boldsymbol{D}_{t_2,correct}\right\}\right] + E\left[\boldsymbol{R}_{yes}\right]\right)$$

$$+ \left(\frac{1-p_t}{2-p_t}\right)\left(E\left[\boldsymbol{D}_{t,correct}\right] + E\left[\boldsymbol{R}_{yes}\right]\right)$$

$$= \left(\frac{p_t}{2-p_t}\right)E[\min\{\boldsymbol{D}_{t_1,correct}, \boldsymbol{D}_{t_2,correct}\}] + \left(\frac{1-p_t}{2-p_t}\right)E[\boldsymbol{D}_{t,correct}] + E[\boldsymbol{R}_{yes}]. \qquad (10)$$

The incorrect response time is given by Case 3 alone. The expected incorrect response time is,

$$\mu_{tt,incorrect} = E[\max\{\boldsymbol{D}_{t_1,incorrect}, \boldsymbol{D}_{t_2,incorrect}\}] + E[\boldsymbol{R}_{no}].$$

Our main focus is on the difference between the expected correct response time for one target and the expected correct response time for two targets. The difference is constructed so that a faster response time to two targets results in a positive difference. Using Equations (8) and (10), the difference is,

$$\mu_{t,correct} - \mu_{tt,correct}$$

$$= \left(E\left[\boldsymbol{D}_{t,correct}\right] + E\left[\boldsymbol{R}_{yes}\right]\right)$$

$$- \left(\left(\frac{p_t}{2-p_t}\right)E\left[\min\left\{\boldsymbol{D}_{t_1,correct}, \boldsymbol{D}_{t_2,correct}\right\}\right] + \left(\frac{1-p_t}{2-p_t}\right)E\left[\boldsymbol{D}_{t,correct}\right]\right.$$

$$\left. + E\left[\boldsymbol{R}_{yes}\right]\right)$$

$$= \left(\frac{2-p_t-1+p_t}{2-p_t}\right)E\left[\boldsymbol{D}_{t,correct}\right] - \left(\frac{p_t}{2-p_t}\right)E\left[\min\left\{\boldsymbol{D}_{t_1,correct}, \boldsymbol{D}_{t_2,correct}\right\}\right]$$

$$= \left(\frac{1}{2-p_t}\right)E\left[\boldsymbol{D}_{t,correct}\right] - \left(\frac{p_t}{2-p_t}\right)E\left[\min\left\{\boldsymbol{D}_{t_1,correct}, \boldsymbol{D}_{t_2,correct}\right\}\right]. \qquad (11)$$

When there are no errors, i.e., $p_t = 1$, the difference is positive, since $E[\boldsymbol{D}_{t,correct}] \geq E[\min\{\boldsymbol{D}_{t_1,correct}, \boldsymbol{D}_{t_2,correct}\}]$. Even with errors ($0 < p_t < 1$), the difference is still positive because $\left(\frac{1}{2-p_t}\right) \geq \left(\frac{p_t}{2-p_t}\right)$ and $E[\boldsymbol{D}_{t,correct}] \geq E[\min\{\boldsymbol{D}_{t_1,correct}, \boldsymbol{D}_{t_2,correct}\}]$. Thus, this parallel model predicts that a correct response time for two targets is faster than the correct response time for one target.

Next consider the difference between the probability of a correct response for two targets and the probability of a correct response for one target. The difference is constructed so that an increase in accuracy for two targets results in a positive difference. Using Equations (7) and (9), the difference is,

$$(2\,p_t - p_t^2) - p_t = p_t - p_t^2 \tag{12}$$

which is greater than or equal to zero because $p_t \geq p_t^2$. Thus, redundant targets improve accuracy.

For completeness, the other predictions of this parallel model are given next.

For a **single distractor condition** d, by definition,

$$p_d = P(\boldsymbol{Z}_d = 1),$$

$$\boldsymbol{T}_{d,correct} = \boldsymbol{D}_{d,correct} + \boldsymbol{R}_{no}, \text{ and}$$

$$\boldsymbol{T}_{d,incorrect} = \boldsymbol{D}_{d,incorrect} + \boldsymbol{R}_{yes}.$$

The expected values are,

$$\mu_{d,correct} = \mathrm{E}[\boldsymbol{D}_{d,correct}] + \mathrm{E}[\boldsymbol{R}_{no}],$$

$$\mu_{d,incorrect} = \mathrm{E}[\boldsymbol{D}_{d,incorrect}] + \mathrm{E}[\boldsymbol{R}_{yes}].$$

For the **two-distractor condition** dd, there are three cases. The first case is when both distractors are processed correctly, in parallel. For this case, the response time is determined by the last component process completed. The probability that Case 1 occurs is,

$$p_{dd,Case1} = P(\boldsymbol{Z}_d = 1 \; and \; \boldsymbol{Z}_d = 1) = p_d^2.$$

The correct processing time for Case 1 is,

$$\boldsymbol{T}_{dd,Case1} = \max\{\boldsymbol{D}_{d_1,correct}, \boldsymbol{D}_{d_2,correct}\} + \boldsymbol{R}_{no},$$

with

$$\mu_{dd,Case1} = \mathrm{E}\left[\max\{\boldsymbol{D}_{d_1,correct}, \boldsymbol{D}_{d_2,correct}\}\right] + \mathrm{E}[\boldsymbol{R}_{no}].$$

The second case is when one distractor is processed incorrectly and the other distractor is processed correctly, in which case the processing is terminated when the distractor is processed incorrectly. The probability that Case 2 occurs is,

$$p_{dd,Case2} = P\left(\boldsymbol{Z}_{d_1} = 0 \text{ and } \boldsymbol{Z}_{d_2} = 1 \text{ or } \boldsymbol{Z}_{d_1} = 1 \text{ and } \boldsymbol{Z}_{d_2} = 0\right) = 2p_d(1 - p_d).$$

The incorrect processing time for Case 2 is,

$$\boldsymbol{T}_{dd,Case2} = \boldsymbol{D}_{d,incorrect} + \boldsymbol{R}_{yes},$$

with

$$\mu_{dd,Case2} = \mathrm{E}\left[\boldsymbol{D}_{d,incorrect}\right] + \mathrm{E}\left[\boldsymbol{R}_{yes}\right].$$

The third case is when both distractors are processed incorrectly. In this case, the response time is determined by the fastest of the two component processes. The probability that Case 3 occurs is,

$$p_{dd,Case3} = P\left(\boldsymbol{Z}_{d_1} = 0 \text{ and } \boldsymbol{Z}_{d_2} = 0\right) = (1 - p_d)^2.$$

The incorrect processing time for Case 3 is,

$$\boldsymbol{T}_{dd,Case3} = \min\{\boldsymbol{D}_{d_1,incorrect}, \boldsymbol{D}_{d_2,incorrect}\} + \boldsymbol{R}_{yes},$$

with

$$\mu_{dd,Case3} = \mathrm{E}\left[\min\{\boldsymbol{D}_{d_1,incorrect}, \boldsymbol{D}_{d_2,incorrect}\}\right] + \mathrm{E}\left[\boldsymbol{R}_{yes}\right].$$

Only the first case yields a correct response, thus

$$p_{dd} = p_{dd,Case1} = p_d^2, \text{ and}$$

$$\boldsymbol{T}_{dd,Case1} = \max\{\boldsymbol{D}_{d_1,correct}, \boldsymbol{D}_{d_2,correct}\} + \boldsymbol{R}_{no},$$

with

$$\mu_{dd,Case1} = \mathrm{E}\left[\max\{\boldsymbol{D}_{d_1,correct}, \boldsymbol{D}_{d_2,correct}\}\right] + \mathrm{E}[\boldsymbol{R}_{no}].$$

The other two cases yield incorrect responses, yielding a mixture distribution. The weight for Case 2 is the fraction of incorrect responses due to Case 2 relative to all incorrect responses,

$$w_{Case2} = \frac{2p_d(1-p_d)}{(2p_d(1-p_d) + (1-p_d)^2)} = \frac{2p_d}{1+p_d}.$$

Similarly, the fraction of incorrect responses due to Case 3 relative to all incorrect responses is,

$$w_{Case3} = \frac{(1-p_d)^2}{(2p_d(1-p_d) + (1-p_d)^2)} = \frac{1-p_d}{1+p_d}.$$

The weighted combination of Cases 2 and 3 yields the expected incorrect response time,

$$\mu_{dd,incorrect} = w_{\mathrm{Case2}} E[\boldsymbol{T}_{dd,Case2}] + w_{\mathrm{Case3}} E[\boldsymbol{T}_{dd,Case3}]$$

$$= \left(\frac{2p_d}{1+p_d}\right)\left(\mathrm{E}\left[\boldsymbol{D}_{d,incorrect}\right] + \mathrm{E}\left[\boldsymbol{R}_{yes}\right]\right)$$

$$+ \left(\frac{1-p_d}{1+p_d}\right)\left(\mathrm{E}\left[\min\{\boldsymbol{D}_{d_1,incorrect}, \boldsymbol{D}_{d_2,incorrect}\}\right] + \mathrm{E}\left[\boldsymbol{R}_{yes}\right]\right)$$

$$= \left(\frac{2p_d}{1+p_d}\right)\left(\mathrm{E}\left[\boldsymbol{D}_{d,incorrect}\right]\right) + \left(\frac{1-p_d}{1+p_d}\right)\left(\mathrm{E}\left[\min\{\boldsymbol{D}_{d_1,incorrect}, \boldsymbol{D}_{d_2,incorrect}\}\right]\right) + \mathrm{E}\left[\boldsymbol{R}_{yes}\right].$$

For the **one target and one distractor condition** td, there are four mutually exclusive cases. The cases are distinguished by whether the target is processed correctly or incorrectly, coupled with whether the distractor is processed correctly or incorrectly. The first case is that the target is processed correctly and the distractor is processed correctly. Here, only the processing of the target determines the response time. The probability that Case 1 occurs is,

$$p_{td,Case1} = p_t p_d$$

$$\boldsymbol{T}_{td,Case1} = \boldsymbol{D}_{t,correct} + \boldsymbol{R}_{yes},$$

with

$$\mu_{td,Case1} = \mathrm{E}\left[\boldsymbol{D}_{t,correct}\right] + \mathrm{E}\left[\boldsymbol{R}_{yes}\right].$$

The second case is that the target is processed correctly and the distractor is processed incorrectly. Now, there is a race between the two processing times. The probability that Case 2 occurs is,

$$p_{td,Case2} = p_t(1 - p_d),$$

$$\boldsymbol{T}_{td,Case2} = \min\left\{\boldsymbol{D}_{t,correct}, \boldsymbol{D}_{d,incorrect}\right\} + \boldsymbol{R}_{yes},$$

with

$$\mu_{td,Case2} = \mathrm{E}\left[\min\left\{\boldsymbol{D}_{t,correct}, \boldsymbol{D}_{d,incorrect}\right\}\right] + \mathrm{E}\left[\boldsymbol{R}_{yes}\right].$$

The third case is that the target is processed incorrectly and the distractor is processed correctly. Now both processes must complete to determine a response. The probability that Case 3 occurs is,

$$p_{td,Case3} = (1 - p_t)p_d,$$

$$\boldsymbol{T}_{td,Case3} = \max\left\{\boldsymbol{D}_{t,incorrect}, \boldsymbol{D}_{d,correct}\right\} + \boldsymbol{R}_{no},$$

with

$$\mu_{td,Case3} = \mathrm{E}\left[\max\left\{\boldsymbol{D}_{t,incorrect}, \boldsymbol{D}_{d,correct}\right\}\right] + \mathrm{E}[\boldsymbol{R}_{no}].$$

The fourth case is that the target is processed incorrectly, and the distractor is processed incorrectly. Here, the distractor processing determines the response time. The probability that Case 4 occurs is,

$$p_{td,Case4} = (1 - p_t)(1 - p_d),$$

$$\boldsymbol{T}_{td,Case4} = \boldsymbol{D}_{d,incorrect} + \boldsymbol{R}_{yes},$$

with

$$\mu_{td,Case4} = \mathrm{E}\left[\boldsymbol{D}_{d,incorrect}\right] + \mathrm{E}\left[\boldsymbol{R}_{yes}\right].$$

The probability of a correct response is achieved through the mutually exclusive cases 1, 2, and 4. It is the probability that the target is processed correctly, as in Case 1 or Case 2, plus the probability that the distractor is processed incorrectly as in Case 4,

$$p_{td} = p_t p_d + p_t(1 - p_d) + (1 - p_t)(1 - p_d)$$

$$= 1 - p_d(1 - p_t).$$

For the three cases that contribute to a correct response, the weights are

$$w_1 = \frac{p_{td,Case1}}{p_{td}} = \frac{p_t p_d}{(1 - p_d(1 - p_t))},$$

$$w_2 = \frac{p_{td,Case2}}{p_{td}} = \frac{p_t(1 - p_d)}{(1 - p_d(1 - p_t))},$$

$$w_4 = \frac{p_{td,Case4}}{p_{td}} = \frac{(1 - p_t)(1 - p_d)}{(1 - p_d(1 - p_t))}.$$

Using these weights, the expected correct response time is,

$$\mu_{td,correct} = w_1 E\left[\boldsymbol{T}_{td,Case1}\right] + w_2 E\left[\boldsymbol{T}_{td,Case2}\right] + w_4 E\left[\boldsymbol{T}_{td,Case4}\right]$$

which does not simplify nicely.

The expected incorrect response time is solely due to Case 3, and is,

$$\mu_{td,Case3} = \mathrm{E}\left[\max\{\boldsymbol{D}_{t,incorrect}, \boldsymbol{D}_{d,correct}\}\right] + \mathrm{E}[\boldsymbol{R}_{no}].$$

The standard self-terminating, unlimited-capacity, parallel model with errors does not allow quantitative predictions, as in the corresponding serial model. The minimum and maximum terms prevent such specific quantitative predictions. Nevertheless, one can make the qualitative prediction that the response time is always reduced with two targets compared to one.

Standard Self-terminating, Fixed-capacity, Parallel Model with Errors

We next consider a fixed-capacity version of our standard self-terminating, parallel model with errors. The term *fixed capacity* is from information theory (Taylor et al., 1967) and means that a constant amount of information is processed from the entire set of stimuli. Thus, assuming equal allocation, half as much information can be extracted from each of two stimuli as can be extracted from one stimulus alone. This idea can be implemented using a sampling process described by Shaw (1980). Such a fixed-capacity model is a special case of a limited-capacity model. The fixed-capacity parallel model provides a useful landmark among the wide range of possible limited-capacity models (e.g., White, Palmer, & Boynton, 2018).

Our goal in analyzing this model is to determine if it predicts that redundant targets have faster response time than single targets, such as found for our unlimited-capacity parallel model. Alternatively, capacity limits might overwhelm the redundancy gain and result in slower response times as found for our serial model. Establishing this prediction helps distinguish serial and parallel models in general.

One can start from the parallel model described in the preceding section by replacing the unlimited-capacity assumption with fixed capacity. Unfortunately, we know of no way to analyze such a distribution-free model. Instead, we define two special cases of the fixed-capacity model and derive numerical predictions. To foreshadow the results, the two versions have quite different predictions.

Version 1: Simple diffusion processes.

To begin, assume all of the model structure of the previous unlimited-capacity parallel model and add a specific stochastic process that generates the responses and response times. In this version, we use a simple diffusion process that has often been applied to response time (e.g., Palmer, Huk, & Shadlen, 2005) and has been elaborated as a theory of visual search (Corbett & Smith, 2020).

Consider n stimuli, where each stimulus can be a target t or a distractor d. For the current task, a diffusion process applied to response time describes the continuous accumulation of relative evidence for the presence of the target t versus the presence of a distractor d. Let the accumulated evidence for stimulus $i, i = 1, \dots, n,$ correspond to a random variable that varies over time $\boldsymbol{U}_i(x)$ where x is time. At time zero, $\boldsymbol{U}_i(0) = 0$. As time increases, evidence is accumulated from a target at a mean rate r_t and from a distractor at a mean rate $-r_d$. (For the details of representing evidence as a signal-to-noise ratio, see Palmer, et al., 2005). The change for the fixed-capacity model is that the rate for this model is reduced by a factor of $1/\sqrt{2}$ relative to an unlimited-capacity model. This is the result of the rate being determined by a set of independent samples that are equally allocated when there are multiple stimuli (Shaw, 1980). With two stimuli, half as many samples can be allocated, as to a single stimulus. This results in twice the variability, or $\sqrt{2}$ the standard deviation of the estimate of the stimulus information. This scales the effect of the stimulus by $1/\sqrt{2}$. For example, if r_t and r_d are 2.0 for the unlimited capacity model, they would be 1.41 for the fixed-capacity model.

The response occurs by evaluating the accumulated evidence for each stimulus. Starting at time zero, all stimuli are unlabeled, and as time increases, the evidence is evaluated to label the stimuli with a positive or negative decision, as follows.

- At time step x, for unlabelled stimuli, evaluate $\boldsymbol{U}_i(x)$:
  - If $\boldsymbol{U}_i(x) > a$, then label stimulus i with a positive decision, and terminate with a "yes" response.
  - If $\boldsymbol{U}_i(x) < -b$, then label stimulus i with a negative decision, and continue.
- If all stimuli are labeled with a negative decision, terminate with a "no" response. Otherwise, increment the time step and repeat.

To complete the definition of the fixed-capacity diffusion model, we fix the coefficient of variability of the residual time to 0.1. This value for the coefficient of variability is motivated by the idea that the residual processes are stereotyped and have a relatively low variance. In contrast, the component processing time from the diffusion process typically has a much larger coefficient of variability of around 0.8 to 1.0. The variability of the total response time is the sum of the variability of the residual time and the component processing time. The coefficient of variability for the total response time found in perceptual tasks varies from 0.1 for strong stimuli (dominated by the residual time) to 0.5 for weak stimuli (contributions from both residual time and component processing time). While useful for specifying the models, this residual time parameter has no effect on the redundant target effect.

To calculate the predictions of this model, we first choose parameter values relative to Experiment 2. All together there are six parameters and they are listed in Table A1.
To do this, the experiment is summarized by the mean proportion correct, mean correct response time, and mean coefficient of variability for the correct response times for the single target and single distractor conditions, averaged over the three tasks (color, lexical, and semantic). This gives six statistics describing the data. Then the six model parameters are estimated that yield the six statistics. The estimated parameter values are listed in Table A1.

Table A1: Parameters for the Diffusion Model

Parameter	Symbol	Experiment 2 Values	Experiment 3 Values
rate for a target	r_t	2.545	1.650
rate for a distractor	r_d	2.200	2.829
upper bound	a	0.593	0.308
lower bound	$-b$	0.425	0.962
mean residual time for a "yes" response	$E[\boldsymbol{R}_{yes}]$	0.628	0.593
mean residual time for a "no" response	$\mathrm{E}[\boldsymbol{R}_{no}]$	0.667	0.616

Importantly, all of this is done with just the single stimulus conditions in Experiment 2. Finally, using these parameters, we calculate the predicted effects of two targets compared to a single target. For this experiment, the fixed-capacity model predicts that two targets are faster and more accurate than a single target (gain of 22 ms and 7.1% correct).

Next consider parameters based on the data from Experiment 3. As above, we determine six parameter values based on six statistics. The estimates are given in Table A1. For these conditions, the fixed-capacity model also predicts that two targets are faster and more accurate than one (gain of 62 ms and 2.4% correct). Thus, these conditions also yield positive redundant target effects. We explored the parameter values for the conditions of each experiment. For all conditions that avoid extreme parameters, the redundant target effect remains positive.

In summary, we evaluated a fixed-capacity parallel model that depends on a simple diffusion process. For all conditions expected in a typical experiment, the fixed-capacity parallel model predicts a positive effect of redundant targets. Thus, the predictions of this version of a fixed-capacity parallel model are distinct from the predictions of negative redundant target effects made by our standard self-terminating serial model.

Version 2: Linear ballistic accumulators.

A different model of response time is the linear ballistic accumulator (LBA) model (Brown and Heathcote, 2008). It differs from the simple diffusion model in several ways. First, the stochastic element is variability in the rate of accumulation from trial to trial, instead of from moment to moment. Second, there are separate accumulators for each response, instead of comparing positive and negative evidence within a single accumulator. Third, there is a "bias" contribution to the rate of accumulating evidence for each accumulator, instead of separate bounds for the net positive and negative evidence. We implemented a particularly simple version of this model. The variability of the rate parameter was described by a Gamma distribution (Terry et al., 2015) to avoid the complications of negative rates that occurred in the original formulation arising from using a Gaussian distribution. In addition, we dropped the feature of variability in the start point. Finally, the predictions from the redundant target conditions were implemented following the derivation in Eidels, Donkin, Brown, & Heathcote, (2010).

For this model, the accumulated evidence for a "yes" or "no" response is in separate accumulators, denoted Y or N, respectively. Such a pair of accumulators exists for each stimulus $i, i = 1, \dots, n$. The accumulated evidence for stimulus i corresponds to two random variables, denoted $\boldsymbol{U}_{Y_i}(x)$ and $\boldsymbol{U}_{N_i}(x)$, where x is time. At time zero, $\boldsymbol{U}_{Y_i}(0) = 0$ and $\boldsymbol{U}_{N_i}(0) = 0$. Evidence is accumulated for a target at a rate of r_{t_Y} and r_{t_N} for each Y and N accumulator, respectively. Similarly, evidence is accumulated for a distractor at a rate of r_{d_Y} and r_{d_N} for each

Y and N accumulator, respectively. These rates can be interpreted in terms of signal component and bias component. Let the signal for a target be $r_{t,signal} = r_{t_Y} - r_{t_N}$ and the bias for a target be $r_{t,bias} = r_{t_N}$. Similarly, let the signal for a distractor be $r_{d,signal} = r_{d_N} - r_{d_Y}$ and the bias for a distractor be $r_{d,bias} = r_{d_Y}$.

This separation of signal and bias components is needed to introduce the idea of fixed capacity. The signal rates determine the accuracy of the response. For example, if $r_{t,signal} = 0$, the responses on the target trials are at chance. To incorporate fixed capacity, the signal rates are reduced by a factor of $1/\sqrt{2}$, just as was done in the diffusion model.

Two additional parameters for this model are: a common bound for all accumulators b, and a common standard deviation for the variability of all rate parameters r_{SD}. These two parameters and all of the rate parameters share a common factor, so one can fix one of these parameters. Hence, we set $r_{SD} = 1$, which is equivalent to making all of these parameters relative to the standard deviation of the rate (see Palmer, et al., 2005). In addition, to further reduce the number of parameters, we set the bound $b = 1$. This is possible because the $r_{t,bias}$ and $r_{d,bias}$ parameters act in a similar way to having separate bound for both accumulators.

A response occurs when the evidence in any "yes" accumulator $\boldsymbol{U}_{Y_i}(x)$ reaches the bound for a "yes" response, or all of the "no" accumulators reach the bound for a "no" response. Starting at time zero, all stimuli are unlabeled, and as time increases, the evidence is evaluated to label the stimuli with a positive or negative decision, as follows.

- At time step x, for unlabelled stimuli, evaluate $\boldsymbol{U}_{Y_i}(x)$: and $\boldsymbol{U}_{N_i}(x)$:
    - If $\boldsymbol{U}_{Y_i}(x) > b$, then label stimulus i with a positive decision, and terminate with a "yes" response.

- If $\boldsymbol{U}_{N_i}(x) > b$, then label stimulus i with a negative decision, and continue.

- If all stimuli are labeled with a negative decision, terminate with a "no" response. Otherwise, increment the time step and repeat.

Finally, to complete the model, we add two residual time parameters: the mean of the residual time for a "yes" response $E[\boldsymbol{R}_{yes}]$ and the mean of the residual time for a "no" response $E[\boldsymbol{R}_{no}]$. Together, there are six parameters and they are listed in Table A2.

Table A2: Parameters for the LBA Model

Parameter	Symbol	Experiment 2 Values	Experiment 3 Values
signal rate for a target	$r_{t,signal}$	2.112	1.257
signal rate for a distractor	$r_{d,signal}$	1.683	2.400
bias rate for a target	$r_{t,bias}$	0.780	1.416
bias rate for a distractor	$r_{d,bias}$	1.240	0.227
mean residual time for a "yes" response	$E[\boldsymbol{R}_{yes}]$	0.447	0.319
mean residual time for a "no" response	$\mathrm{E}[\boldsymbol{R}_{no}]$	0.449	0.502

As with the diffusion model, we numerically solve for these parameters based on six statistics from the single target and single distractor conditions of our experiments. The six parameters with numerical values are listed in Table A2. From these parameters, the predicted redundant target effects were calculated. For Experiment 2, there was a positive redundant target effect of 3 ms on response time, and 9.3% on accuracy. In contrast, for Experiment 3, there was a negative redundant target effect of −37 ms on response time. This was accompanied by a positive 2.6% effect on accuracy. The important new result is that this model can predict a negative redundant target effect.

Why do these two versions of the fixed-capacity, parallel model differ regarding the sign of the redundant target effect? We suspect an important factor is the degree of variability in the component processing time for each stimulus. For the diffusion model, this variability is quite high, with a coefficient of variability around 1.0. In contrast, for the linear ballistic accumulator model, the variability is determined by the set of parameters. For Experiment 3, the coefficient of variability of the component time for the selected parameters was only 0.25. Such lower variability results in a smaller redundant target effect that can be overcome by the increase in processing time due to fixed capacity. This results in a negative redundant target effect.

Summary

We have presented two version of the standard self-terminating, fixed-capacity, parallel model with errors. One is based on a diffusion process and the other based on the linear ballistic accumulator. For parameters based on the single stimulus conditions of Experiment 3, these models make quite different predictions about the redundant target effect on response time: one positive and the other negative. Thus, this general class of model makes no prediction about whether the redundant target effect for response time is positive or negative.

Details about Calculating Predictions of Redundant Target Effects

In the introduction of the article, Figure 2 showed typical range of predictions of redundant target effects for our three landmark models. Here, we describe the details of how we made these predictions.

The predictions for the serial model are relatively simple to calculate, based on Equation (5). The only factors affecting the predictions are the mean component processing time for errors on target trials (misses) and the proportion of correct responses for a single target. For all predictions and all models, we kept the mean correct response time fixed at 800 ms and percent errors at 5%. To set the mean component processing time for errors, we first assumed a residual time of 100 ms. Then, for the upper limit, we assumed equal component processing time for correct (hit) and error (misses) responses. For the lower limit, the mean component processing time for errors was assumed to be twice as long as the mean component processing time for correct responses. These predictions span the range expected in a typical experiment.

For our unlimited-capacity, parallel model, there are no direct numerical predictions from Equation (10). Instead, we rely on the two special cases we have already developed: the diffusion model and the linear ballistic accumulator model. For the upper limit, we used the diffusion model with parameters that for the single stimulus trials yield 5% errors, a mean response time of 800 ms and coefficient of variability of 0.2. These constraints were used for both target and distractor trials. The result is a relatively large redundant target effect. For the lower limit, we used the linear ballistic accumulator model in a similar way. To minimize the redundant target effects, we reduced the coefficient of variability to 0.1. This yielded a relatively small redundant target effect. While these predictions are not tight bounds, they illustrate the range of typical effects.

Lastly, consider the predictions of our fixed-capacity, parallel model. As with our unlimited-capacity model, we rely on the two special cases with all of the same constraints. For this model, the predictions now span a range that includes both positive and negative redundant target effects.